PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 71 SEPTEMBER, 1945 No. 7

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

Printed in the United States of America

Current Papers and Discussions

	Published	Discussion closes
Vanoni, Vito A. Transportation of Suspended Sediment by Water	June, 1944	
Discussion in Dec., 1944, Feb., Mar., June, Sept., 1945		Closed
Hickox, G. H. Evaporation from a Free Water Surface	Oct., 1944	
Discussion in Feb., Mar., Apr., May, Sept., 1945		Closed
Johnson, Lee H., Jr. Analysis of Statically Indeterminate Structures Using Reduced Equations	Nov., 1944	
Discussion in Feb., May, June, 1945		Closed*
Bull, Anders. Stresses in the Linings of Shield-Driven Tunnels	Nov., 1944	
Discussion in Jan., Feb., May, June, Sept., 1945		Oct. 1, 1945
Kalinske, A. A. Conversion of Kinetic to Potential Energy in Flow Expansions	Dec., 1944	
Discussion in Feb., May, June, Sept., 1945		Oct. 1, 1945
Osgood, William R. Column Formulas	Dec., 1944	
Discussion in Feb., Mar., May, June, 1945		Oct. 1, 1945
Blanks, R. F., and Meissner, H. S. Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction	Jan., 1945	
Discussion in Sept., 1945		Oct. 1, 1945
DeYoung, Isaac. A Plan for a Movable Dam	Feb., 1945	
Discussion in May, June, 1945		Oct. 1, 1945
Powell, Ralph W. Flow in a Channel of Definite Roughness	Dec., 1944	
Discussion in June, 1945		Oct. 1, 1945
Bennett, Preston T. The Effect of Blankets on Seepage Through Pervious Foundations	Jan., 1945	
Discussions in Mar., Apr., June, 1945		Oct. 1, 1945
Conkling, Harold. Utilization of Ground-Water Storage in Stream System Development	Jan., 1945	
Discussion in Mar., Apr., May, June, Sept., 1945		Oct. 1, 1945
Weiss, Camillo. Interrelation of Certain Structural Concepts	Jan., 1945	
Discussion in May, June, Sept., 1945		Oct. 1, 1945
Bakhmeteff, Boris A., and Allan, William. The Mechanism of Energy Loss in Fluid Friction	Feb., 1945	
Discussion in Sept., 1945		Nov. 1, 1945
Hayes, Harry. Elimination of Cross Connections in Los Angeles, Calif.	Mar., 1945	
Discussion in May, 1945		Nov. 1, 1945
Eliel, Leon T. Economics of Photogrammetry	Mar., 1945	
Discussion in June, 1945		Nov. 1, 1945
Nelidov, I. M. Determination of Pressures within a Siphon Spillway	Mar., 1945	Nov. 1, 1945
Genter, A. L. Computing Coagulant Requirements in Sludge Conditioning	Mar., 1945	
Discussion in June, 1945		Nov. 1, 1945
Mullis, Ira B. Design of Roadbeds	Apr., 1945	Nov. 1, 1945
Harrold, Lloyd L. Flow from Drainage Basins Determined by Short-Term Records	Apr., 1945	Nov. 1, 1945
Camp, Thomas R. Sedimentation and the Design of Settling Tanks	Apr., 1945	
Discussion in September, 1945		Nov. 1, 1945
Stevens, J. C. Future of Lake Mead and Elephant Butte Reservoir	May, 1945	Nov. 1, 1945
Brumfield, R. C. Moving Loads on Restrained Beams and Frames	May, 1945	Nov. 1, 1945
Eichler, John Oran, and Eves, Howard W. Mathematical Examination of the Highway Transition Spiral	May, 1945	Nov. 1, 1945
Report: Water Supply Engineering: Report of the Committee of the Sanitary Engineering Division for the Two Years Ending December 31, 1944	May, 1945	Nov. 1, 1945
Johnston, Milan A. Effects of Radiant Heat on Reinforced-Concrete Rigid Frames	June, 1945	Nov. 1, 1945
Ehasz, F. L. Structural Skew Plates	June, 1945	Nov. 1, 1945
Laverty, Finley B. Correlating Flood Control and Water Supply, Los Angeles Coastal Plain	June, 1945	Nov. 1, 1945

Note.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR SEPTEMBER, 1945

PAPERS

DISCUSSIONS

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

The Society is not responsible for any statement made or opinion expressed in its publications

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Secretary, Committee on Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

CAVITATION IN HYDRAULIC STRUCTURES
A SYMPOSIUM

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by February 1, 1946.

NATURE OF CAVITATION

By John K. Vennard,[1] Assoc. M. Am. Soc. C. E.

Foreword

As part of the program of the Hydraulic Research Committee of the Hydraulics Division, a Subcommittee on Cavitation in Hydraulic Structures was appointed in the summer of 1942. Its first objective was to summarize the facts of cavitation which are pertinent to civil engineering problems and to bring to the attention of the profession certain experiences with the phenomenon. It was decided that this objective could best be attained by the presentation of a Symposium where there would be opportunity for others, by discussion, to report their experiences and thus bring additional facts to light. In this way the store of knowledge could be increased and assembled in convenient form for civil engineering use. The Symposium developed logically into a single paper containing a summary of facts and theories, followed by three papers on practical experiences with cavitation.

After current knowledge of cavitation has been summarized by the Symposium and ensuing discussions, the committee intends to stimulate research activity on the unanswered questions and unsolved problems with the ultimate objective of obtaining, for the designer, sufficient information for cavitation-free designs to be produced with certainty.

Introduction

Cavitation first became an engineering problem about 1900 when, with the use of the steam turbine drive for ship propulsion (which resulted in higher propeller speeds), marine engineers noticed losses of efficiency and destruction of propeller materials. Shortly afterward, mechanical engineers in the hydraulic machinery field encountered the phenomenon as speeds of turbines and pumps were increased. With the design of higher hydraulic structures and with their resulting higher velocities, the civil engineer has become increasingly involved with the cavitation problem.

The generally accepted word for the destruction and subsequent erosion of materials by cavitation action is "pitting." Although the operator frequently refers to the erosion of blades, conduits, piers, liners, etc., as "cavitation," this use of the word is to be discouraged. In brief, the cavity occurs in the liquid, the pitting in the solid boundary. Although the damaging or "pitting" of structural materials by cavitation has been variously attributed to chemical action, electrochemical action (corrosion), high tension in the water, etc., present evidence indicates conclusively that the destructive action is essentially a mechanical one, caused by the impact of liquid masses on the surface of the material. The objective of all designers is to gain sufficient knowledge of the

[1] Asst. Prof., Fluid Mechanics, New York Univ., New York, N. Y.

nature and occurrence of cavitation to prevent it or render it harmless in new designs.

Notation.—The letter symbols, used in this Symposium, conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942. In each paper definitions are introduced where a symbol is first mentioned.

Occurrence of Cavitation

Because liquids encountered in engineering practice cannot expand and cannot support tension stress, cavities will form in them wherever the absolute pressure falls to (or close to) the vapor pressure of the liquid. Two of the factors contributing to pressure reductions may be seen, from the Bernoulli equation for a frictionless liquid—

$$\frac{p}{\gamma} + \frac{V^2}{2\,g} + Z = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

—to be increments of velocity, V, and of elevation, Z. (The remaining symbols in Eq. 1 are defined as follows: p is the pressure per unit area; γ is the specific weight; and g is the acceleration due to gravity.)

Considering the effect of elevation it may be concluded, for example, that (*a*) cavitation is more likely to appear at the top of a conduit than at the bottom, (*b*) higher setting of an hydraulic turbine above tailwater will increase the tendency for cavitation, and (*c*) marine propellers on surface vessels are more susceptible to cavitation than the propellers of submarines (when submerged).

The production of pressure reduction by increase of velocity is familiar to the engineer and usually results from constriction of a passage as, for example, in a venturi meter or aspirator nozzle; the tendency for cavitation to occur in regions of high velocity is obvious.

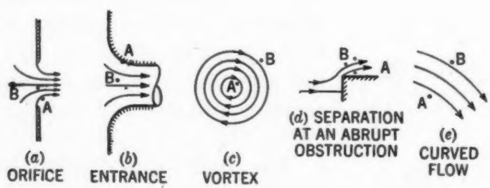


Fig. 1.—Pressure Reductions toward the Centers of Flow Curvatures

A third, and probably the most important (yet most obscure and unpredictable), factor contributing to pressure reduction and cavitation does not appear in Eq. 1. This is flow curvature, the essential facts of which are shown in Fig. 1, points A denoting the areas of low pressure and points B the areas of high pressure. (The effects of curved flow, of course, may be included in the Bernoulli equation if one continues to use the frictionless liquid as a basis for his reasoning. Since flow curvatures are invariably caused, or accompanied,

by separation, eddies, vortices, boundary layer phenomena, etc.—all of which are the result of viscous action—the assumption of a frictionless fluid is not a reasonable one.) Cavitation caused by the pressure reduction at local flow curvatures is probably more prevalent in engineering practice than any other type. These flow curvatures may exist on a boundary surface of relatively easy curvature such as a bellmouth conduit entrance, on the blades of a turbine or propeller, or at abrupt corners which produce flow separation such as gate slots, partly opened valves, or offsets in a boundary surface due to poor alinement. Furthermore, localized regions of high velocity and low pressure exist in the sharply curved flow at the centers of vortices and eddies and produce an irregular and unpredictable type of cavitation.

The relative motion of solid and liquid so necessary to the production of low pressure may be obtained by moving the solid as well as the liquid. Cavitation may be produced this way in reciprocating or vibrating devices such as plunger pumps and submarine signaling diaphragms, and in the vibratory test apparatus used by H. Peters (1).[2] This ingenious device, illustrated subsequently in Fig. 8, has proved to be an efficient means of studying cavitation under controlled conditions and producing accelerated pitting. It consists of a vertical nickel tube which is oscillated longitudinally at 6,500 cycles per sec by magnetostriction. The specimen to be tested is attached to the lower end of the rod which is immersed in a beaker of liquid. Oscillation of the rod causes cavitation in contact with the bottom of the specimen and produces pitting of the type shown subsequently in Fig. 12.

Nature of Cavitation

Cavities produced by low pressures in a flowing liquid prove to be unstable and periodically detach themselves from their point of formation. They may then be swept rapidly downstream into a region of higher pressure where they are suddenly collapsed by the surrounding liquid. As the inward rushing liquid finally fills the cavity, the momentum of the liquid is reduced to zero in an almost infinitesimal time. If the point of collapse is on a solid boundary, the pressure over an infinitesimal area becomes enormous and capable of denting or pitting the solid material within that area.

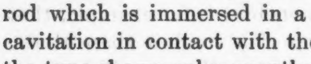


Fig. 2.—Pressure Variations in a Convergent-Divergent Passage with and without Cavitation

For study in its simplest form in the laboratory, cavitation may be produced in a convergent-divergent passage with transparent walls and the foregoing facts may be deduced from observations. In the apparatus of Fig. 2 the conventional hydraulic grade line A will be produced for a small valve opening and small flow rate. At a larger valve opening the higher velocity at the throat of the constricted passage produces an absolute pressure equal to the

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography, which appears as the last unit of the Symposium.

vapor pressure p_v of the liquid. Then a frothy region appears extending from the throat of the constriction into the divergent tube and the hydraulic grade line B will be obtained. Although the frothy region appears steady to the naked eye, high-speed motion pictures indicate it to be forming and reforming many times every second. The cavities form at the throat, extend themselves downstream, are torn away from the throat, and disappear near region B where the pressure gradient is high. Near B the observation window will be found after a short time to be pitted as if from delicate taps by a sharp instrument. Fig. 3

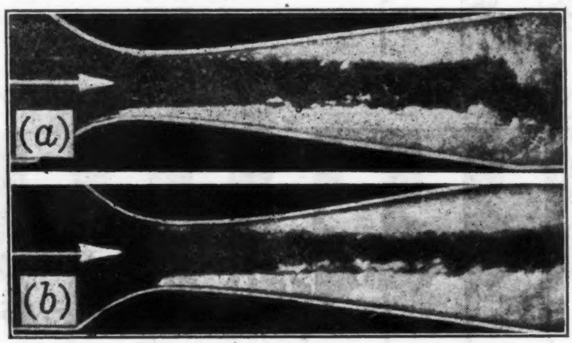


FIG. 3.—CAVITATION IN A CONVERGENT-DIVERGENT PASSAGE

shows cavitation in a divergent passage as photographed by H. Föttinger. Fig. 4(*a*) shows schematically the creation, travel, and collapse of the cavity, and Fig. 4(*b*) shows the same series of events (cavity in black), as recorded by the high-speed camera at the Massachusetts Institute of Technology, at Cambridge, Mass.

It is not implied that all cavitation assumes the form outlined in Fig. 4 but from such tests its essential nature may be deduced and the parts of the process discussed separately. Evidently the cavitation mechanism is composed of four parts: (*a*) The formation of the cavity; (*b*) its travel; (*c*) its final collapse; and (*d*) the consequences of its collapse. Since pitting of adjacent materials is one of the consequences of cavity collapse, a brief statement will be made on the pitting mechanism and on remedies that have proved successful in arresting it. In addition to the foregoing, certain luminous phenomena have been observed in the high-velocity flows on spillway aprons and in the flows downstream from the outlets of draft tubes. Since similar observations have been made on flows both with and without cavitation, it appears that these luminous phenomena are not one of the consequences of cavitation (2).

FORMATION OF THE CAVITY

Because cavities are formed at regions of low pressure and because air or other gases dissolved in liquids come out of solution in these regions it would be expected that the pressures within the cavities are larger than the vapor pressure of the liquid by the partial pressure of the air within the cavity. Measurements

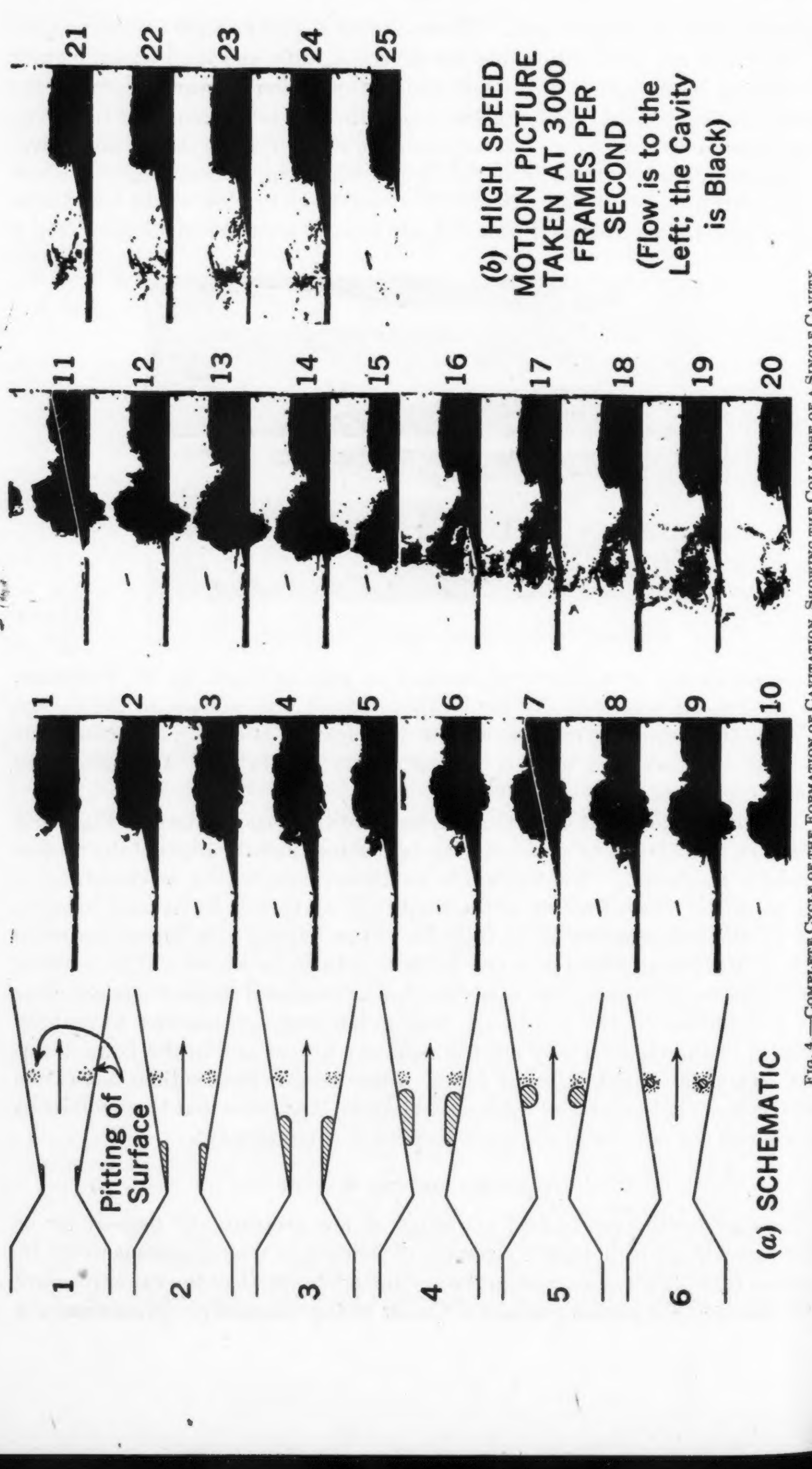


FIG. 4.—COMPLETE CYCLE OF THE FORMATION OF CAVITATION, SHOWING THE COLLAPSE OF A SINGLE CAVITY

(3) have indicated this to be the case and have also shown that the excess of cavity pressure over vapor pressure increases with the air content of the liquid.

When cavities are formed by separation from a divergent boundary, as illustrated in Fig. 4, they have a fairly regular frequency which is approximately proportional to the velocity of flow and inversely proportional to the length of the cavity (3). The detachment of a given cavity from the boundary wall is probably caused by an instantaneous rise of pressure due to the collapse of the preceding cavity.

A cavity caused by a vortex (Fig. 5) does not form and collapse in the manner demonstrated in Fig. 4. In fact, if the flow were perfectly steady it would be difficult to see how any collapse would occur. However, the turbulent

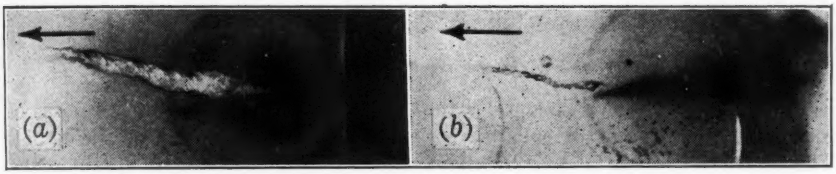


FIG. 5.—CAVITATION CAUSED BY A VORTEX (VELOCITY OF APPROACH TO BLADE, 9 FT PER SEC): (a) GAP, 1/16 IN.; ANGLE OF ATTACK, + 2.5°; AND (b) GAP, 1/4 IN.; ANGLE OF ATTACK, − 2.5°

nature of the flow causes whipping of the unattached end of the vortex, which causes small portions of the major cavity to be detached and to collapse in a region of higher pressure. In Fig. 5 there is a gap between the end of the blade and the glass plate through which the pictures were taken. Upward flow occurs through the gap and produces a vortex similar to the tip vortex at the end of an airplane wing.

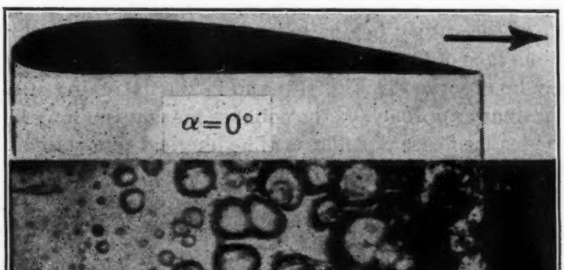


FIG. 6.—CAVITATION BUBBLES FORMING ON THE TOP OF A BLADE

When eddies that are shed from boundary irregularities, or are caused by the mixing of jets, enter a low-pressure region the pressures at their centers may be low enough to produce a cavity that will then continue to move with the flow and collapse farther downstream. Other types of cavities apparently form as small bubbles, possibly starting with a tiny air bubble as a nucleus. Such bubbles have been observed (see Fig. 6) in the flowing fluid (4), whereas

others, moving very slowly, remain in contact with solid boundaries of the flow (5). Such bubbles have been observed to expand in a region of low pressure and disappear when they enter a region of higher pressure. Evidently surface tension will affect cavitation to the extent that such small bubbles play an effective rôle in the phenomenon (6).

Although the thermodynamics of cavity formation is not known quantitatively (7), a lack of equilibrium may be safely assumed except in the case of a cavity at the center of a fixed vortex. As a cavity forms and rapidly increases in size, vaporization of the liquid will occur and gases dissolved in the liquid will come out of solution into the cavity. However, due to the turbulence that always accompanies cavitation, the speed of vaporization and of release of dissolved gases is quite unpredictable. Little is known of the process except that it occurs with great rapidity.

Motion of the Cavity

It has been observed generally that the cavity proceeds from its point of formation to its point of collapse with a speed less than that of the liquid. The cavities of Fig. 4(*b*) move at about one half the velocity of the liquid at the throat of the constriction. In Fig. 6 the cavitation bubbles, approximately spherical in shape, can be seen forming and disappearing. They are moving at approximately 15 ft per sec, whereas the stream speed is approximately 22 ft per sec. However, in the case of free small bubbles and small eddies having cavities at their centers, the speed of the cavity is probably about equal to that of the liquid. On the other hand, small bubbles attached to boundary walls have been observed to move at speeds very much less than the speed of the liquid (6).

Collapse of the Cavity

Destruction of the cavity begins when the cavity is in motion and results from the upstream face of the cavity moving more rapidly than the downstream face. This may be observed in Fig. 4(*b*) and in the flattening of the bubbles of Fig. 6. A preliminary approach to the collapse mechanism may be obtained by considering the idealized case of collapse of a spherical cavity in a mass of liquid which is at rest. By assuming that the cavity remained spherical as it collapsed, Lord Rayleigh (8) computed the pressure within the cavity to be 68 tons per sq in. and 765 tons per sq in. when its diameter had been reduced to 1/20 and 1/100, respectively, of its original diameter. C. A. Parsons and S. S. Cook (9) also showed experimentally that enormous pressures may be developed locally when water rushes into a void space containing only water vapor. At first the bubble collapse theory was accepted as a complete explanation for the production of intense local pressures and consequent destruction of materials, but further evidence indicated it to be only a partial answer. Such evidence consisted of high-speed pictures like those of Figs. 4(*b*) and 6, destruction of materials by water jets, and the realization that it was unlikely for a spherical bubble to exist in a flow and inconceivable that it would remain spherical as it collapsed. The views in Fig. 4(*b*) show the cavity to be of irregular shape and to contain droplets of liquid as well as liquid vapor. Tests

made by E. Honegger (10) and T. F. Hengstenberg (11) by moving elements of solid material through small water jets at high speed produced destruction of the material and indicated that appreciable pressures could be produced by impact of small liquid masses on solid surfaces.

From the foregoing facts it appears that the intense local pressures developed at the point of collapse are probably due to a combination of the foregoing phenomena. As collapse of the cavity occurs at a boundary, droplets may be shot at the solid surface with speeds sufficient to produce deformation and eventual destruction, or irregularities in the cavity walls may produce intense local pressures when the liquid strikes the solid material at the high speeds produced at collapse. Although the collapse of the cavity is the main cause of impact between liquid masses and solid boundaries the exact details of the impact mechanism may be quite different in successive collapses.

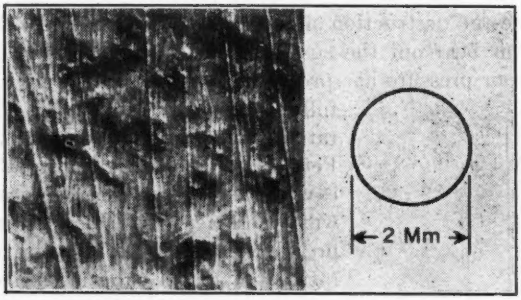


FIG. 7.—SMALL PITS (DENTS) PRODUCED ON A SMOOTH BRASS PLATE AFTER FIVE HOURS OF EXPOSURE TO CAVITATION

Experimental measurement of the pressures produced locally at cavitation collapse has proved an exceedingly difficult problem because these pressures are not produced at exactly the same point at every collapse. Even assuming a perfect measuring instrument, it is still difficult to place the instrument at the exact point where collapse occurs. J. Ackeret (12) has approached the problem by using a piezoelectric crystal 2 mm in diameter (placed in the boundary wall) and has measured local pressures of more than 200 lb per sq in. in a region where the average pressure was only 60 lb per sq in. That local pressures at the collapse point are higher than this and that they exist on a much smaller area may be inferred from Fig. 7 by comparing the size of a dent caused by cavitation on a brass plate with the 2-mm circle. The existence of high pressures in the region of collapse was also convincingly demonstrated by some tests at Massachusetts Institute of Technology (3), in which short lead tubes, having one end closed, were filled with water and placed with their open ends in the region of cavity collapse. The tubes were stretched quickly by the intermittent high pressures and soon failed in tension. The magnitude of collapse pressures cannot be deduced from such tests, of course, but the existence of high pressure is proved beyond doubt.

The thermodynamic processes in cavity collapse consist of high-speed compression of the gases (mostly air) in the cavity and the condensation of the

liquid vapor. In spite of this rapid compression and condensation, high local temperatures probably do not result because of the large water masses available to carry away heat generated by compression. Furthermore, it should be noted that the expansion and vaporization at the formation of the cavity will tend to cool the surroundings so that the heat generated during collapse will tend to restore the temperature to its original value rather than raise it appreciably above that value. Thus it appears very doubtful that thermal effects can play any appreciable part in the pitting of materials by cavitation.

Although local temperature rise appears to be of little consequence in cavity collapse, vapor pressure and air content play a significant part. Higher partial pressures of air and vapor in the collapsing cavity would be expected to offer resistance to the compression of the cavity by slowing condensation of vapor and reabsorption of air by the surrounding liquid. This in turn could be expected to produce a cushioning effect on the collapse, causing local pressures to be less intense and destruction of materials to be less rapid. Laboratory and field observations bear out the cushioning effect of increased air content but variation of vapor pressure has produced various trends. In a series of carefully controlled tests in the vibratory apparatus, using water of constant air content, H. Peters and B. G. Rightmire (13) showed that cavitation severity may increase or decrease with increments of temperature (and vapor pressure). (In Fig. 8, illustrating this apparatus, the nickel tube is oscillated longitudinally by magnetostriction.) However, J. M. Mousson, M. Am. Soc. C. E. (14), reports swifter destruction of materials at higher temperatures in tests using a venturi type apparatus, and turbine operators have generally observed more destruction of blades and parts in the summer when temperature and vapor pressure are high and air content low than in the winter when opposite conditions exist. From the evidence available it appears that no general conclusions may be drawn as to the effect of vapor pressure on cavitation severity. Evidently the rôle that vapor pressure plays in cavity collapse is something more complicated than a cushioning effect.

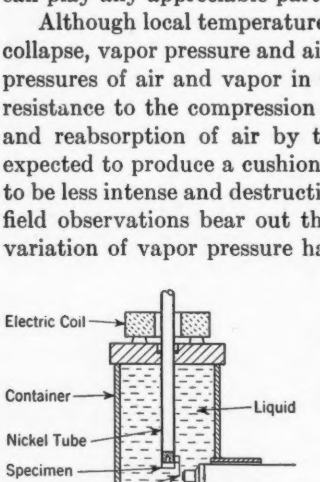


FIG. 8.—APPARATUS FOR PRODUCING CAVITATION BY THE VIBRATORY METHOD

From the random paths that cavities follow before they collapse it is evident that scattering of their points of collapse over a sometimes sizable region may be expected. Since this region is three dimensional, however, it appears likely that most cavities will collapse within the free flow and out of contact with the confining boundary surface. The question naturally arises whether cavities must contact the boundary surface as they collapse in order to damage this surface. As cavities completely surrounded by liquid collapse in the flow, compression waves will be sent out in all directions and will strike the solid boundaries confining the flow. Although it appears unlikely that such pressure waves could damage the boundary material, T. C. Poulter (15) and J. Ackeret and P. deHaller (16) report the damaging of a metal surface in a liquid by the

creation of high-frequency pressure waves in the liquid. If such pressure waves are caused by ordinary cavity collapse, destruction of material at a distance from the collapse point appears possible. Whether collapse of cavities in the flow can destroy boundary surfaces by the action of the pressure waves is still an open question but pressure waves may endanger a structure or machine by setting up forced vibrations. The pressure wave in traveling upstream into the low-pressure region also appears to be instrumental in detaching the succeeding cavity.

Damage from Cavitation, or Pitting

The harmful effects of cavitation are: (*a*) The pitting of the solid boundaries confining the flow; (*b*) the reduction in efficiency of machines and water passages; and (*c*) vibrations in the structure or machine caused by the periodic nature of cavity collapse. Thus, the designer of hydraulic structures may pay a three-fold penalty for allowing cavitation to occur, or he may reap a triple dividend by preventing the occurrence of cavitation. Since a discussion of vibrations and efficiency losses caused by cavitation would lead too far afield, pitting alone will be discussed in this paper. Figs. 9 to 12 illustrate the

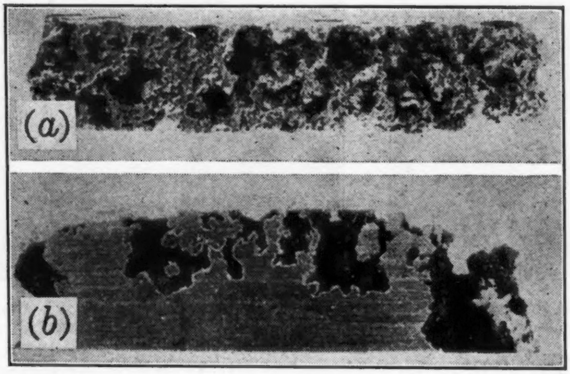


Fig. 9.—Example of Severely Pitted Cast Iron: (*a*) Surface Appearance; and (*b*) Cross Section

typical destruction of metals by cavitation. Fig. 9 was reported by Professor Ackeret (12), Fig. 10 by the Baldwin Southwark Company, Fig. 11 by the National Electric Light Association, and Fig. 12 by the Massachusetts Institute of Technology.

When a cavity collapses adjacent to a solid surface, or a droplet of liquid strikes the surface at high speed, the mechanical action is similar to striking the surface with a small ball-peen hammer. Indeed the dent that results in a malleable material appears just like the result of a hammer blow (see Fig. 7). The action of the collapsing cavity on the material is then primarily mechanical and thermodynamic or electrochemical (corrosion) effects play only a minor rôle. Although it was widely contended some time ago that pitting was primarily a corrosive action, this belief was destroyed by Professor Föttinger (18) when he obtained pitting on the glass walls of a venturi type cavitation

apparatus. However, this is not to claim that corrosive action plays no part at all, because it is well known that metals fatigue more rapidly in the presence of corrosive action.

The pitting of materials by cavitation is primarily a fatiguing action in which the surface skin of the boundary is continuously hammered by millions of tiny blows until it cracks and chips off. It has been observed generally that

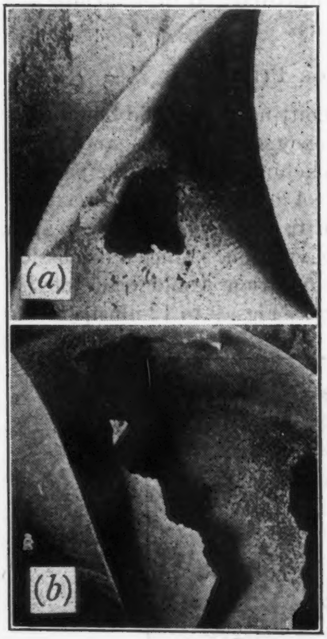


FIG. 10.—SEVERE PITTING OF AN HYDRAULIC TURBINE RUNNER

FIG. 11.—SEVERE PITTING OF A CENTRIFUGAL PUMP IMPELLER

surface finish has an effect on the speed of pitting due to cavitation and on the rapidity of destruction of metal by water jets. Rough surfaces are invariably destroyed more rapidly than surfaces of smooth finish. Thus, as the smooth surface of any material is worn away, the pitting process accelerates and very rapid destruction usually results. In some cases, however, the pitting has stopped of itself, apparently due to a water cushion covering the eroded region and preventing direct contact of collapse point and solid material.

The precise mechanism of the accelerated destruction of rough surfaces is not known, but higher stress concentrations on irregular surfaces undoubtedly exist and should contribute decidedly to the process. Other effects may be at work in the case of very rough or cracked surfaces, however. For example, it is possible that the high pressure nucleus of the collapse cavity (possibly a tiny air bubble) is driven into fissures in the material and explodes when the pressure drops in the surrounding region. Another possibility is that the spaces at the inner ends of the cracks in the material act as cavities themselves

and collapse with destructive force when local pressures at their outer ends are increased by cavities collapsing in the flow. Still another possible explanation is the high pressure developed at the inner end of the fissure by reflection of a pressure wave entering the fissure (similar to the action of water hammer at the closed end of a pipe line). The intermolecular penetration of a high-

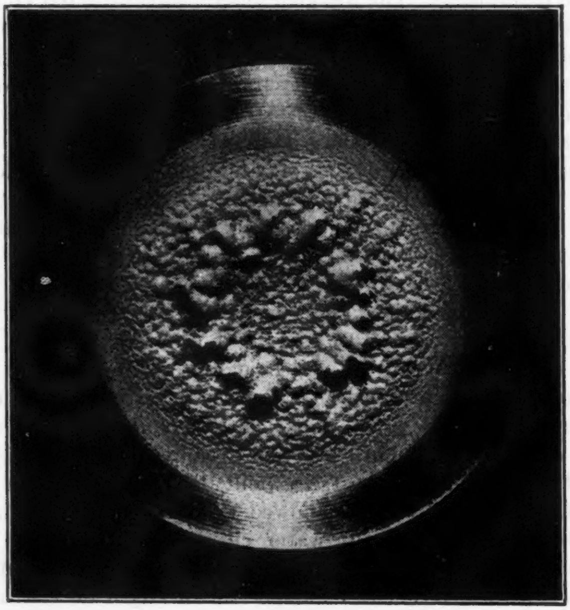


Fig. 12.—Typical Pitting Produced by the Vibratory Method

pressure liquid into a solid has been shown by Mr. Poulter (15) to be capable of destroying the solid upon release of the pressure. Such destruction, however, is dependent upon the application of pressure over a relatively long time. Because of the almost instantaneous existence of the high collapse pressure, it does not appear likely that this effect is operative in contributing to the pitting caused by cavitation.

Evidence is also available that cracks in the solid boundary do not accelerate the pitting process. Mr. Mousson (19*e*) has observed that the regions in and around the check cracks in the welding of a repaired turbine runner show no more pitting than the smooth surface material. He has also reported that destruction or weakening of the material beneath the surface is possible before cracks appear in the surface, by transmission of the effects of the blows on the surface.

Pitting of solid material suspended in a liquid, in which high-frequency pressure waves are created without cavitation, has been obtained by Mr. Poulter (15) and may be explained by the induced vibrations in the solid surfaces causing destructively high local stresses. If solid boundaries are actually destroyed in cavitation by cavities collapsing out of contact with the boundaries, this phenomenon offers an explanation of such pitting.

Remedies for Pitting

The most effective means of preventing pitting in new structures is the elimination of the cavitation that causes it. However, to "design cavitation out" of structures is not possible by design office calculations only. Cavitation can be prevented only through intelligent cooperation of the design office and the hydraulic laboratory. The designer should have in mind continually the facts that sharp curvatures, abrupt corners, or any combination of circumstances which produce flow curvature, vortices, eddies, separation, or high local velocities, are all conducive to cavitation and, like the aeronautical designer, he should make every effort to "streamline" his designs with contours of easy curvature. Even with these facts at hand, the design office cannot produce designs free from the danger of cavitation (unless such designs are so conservative that their costs are prohibitive) without the assistance of the laboratory. This results from the impossibility of using the foregoing principles quantitatively for the precise prediction of hydraulic phenomena.

In completed structures where cavitation and pitting occur and drastic modifications are not possible, various methods have been used successfully to arrest the pitting process or to prevent it after repairs have been made. Replacement of eroded parts by a tougher, more resistant material (such as the use of steel liners in conduits in the region where pitting has occurred in the concrete) has proved successful. This has its counterpart in the hydraulic turbine field where it is standard practice to chip out the damaged material and replace it with a tougher substance built up by welding. In some structures the pitting process has been allowed to continue until it arrests itself, presumably by the creation of a water cushion, as mentioned previously. Although the latter method may have little appeal, it is sometimes effective where erosion does not seriously endanger the strength of a structure or its working parts.

Admission of air to the low-pressure regions has proved effective in both hydraulic structures and hydraulic machines. Higher air content of the water has been seen to cushion the cavity collapse and reduce its destructive effects, and air introduced well upstream from the cavitation region will contribute to this process. However, the effect of introducing air at the cavitation region is to "break the vacuum" there, thus raising the pressure and tending to eliminate true cavitation. The air introduced in this manner presumably forms large bubbles which are compressed downstream in the high-pressure region but are prevented from collapsing by the cushioning effect of the large quantity of air, which cannot be absorbed by the water upon compression of the bubble.

Laboratory Testing for Cavitation

New designs may be tested for cavitation in the laboratory by suitable hydraulic measurements on a model of the new design. For cavitation testing of models there are two techniques which are commonly used: (1) The operation of the model within a vacuum tank; and (2) the operation of the model at or near atmospheric pressure conditions in the usual way. The latter method has been used successfully with water flowing in the model, and more recently with air as the fluid flowing.

The testing of models for cavitation near atmospheric conditions involves the measurement of static pressures at numerous points in the critical region so that the pressure distribution is well known. Cavitation is not produced in the model—if water is used the velocities are not high enough to cause cavitation; and if air is used, cavitation is not possible because of the capability of the fluid to expand and fill any cavities that tend to form. However, by converting the measured model pressures to their corresponding prototype pressures by the laws of dynamic similarity, the model engineer may predict whether the prototype will have cavitation or not. The exact form of the cavitation cannot be predicted from such measurements and it is conceivable that a vortex in the flow which might produce cavitation in the prototype would be missed completely in a model that was equipped with wall piezometer openings only. In spite of this slight element of doubt, this method of testing has proved reliable in the past and it has been adapted successfully of late to the measurement of the intermittent cavitation pressures produced in highly turbulent flows, notably in the flow occurring around baffle piers. In situations like this, flow may be entirely unsteady locally, whereas the over-all flow is a perfectly steady one. At any given point, pressures may be low enough to produce cavitation at one instant and high enough to prevent cavitation at the next instant, thus producing a series of intermittent cavities or "flashes." Obviously such high-frequency pressure fluctuations could not be recorded by a piezometer column because of the inertia of the column; so the U. S. Waterways Experiment Station developed (20) an electrical recording pressure measuring device that traces the pressure fluctuations and thus may be used to predict at any point the percentage of time that cavitation pressures will occur. Assuming that all cavitation cannot be eliminated from abrupt obstructions like baffle piers (which for their very purpose must be unstreamlined), such measurements are useful in correcting designs to eliminate the cavitation pressures which exist continually over wide areas and are thus most likely to produce the regular cavitation that would be damaging to the structure.

Vacuum-tank testing of models for cavitation is much more complicated than open testing but has been shown by H. A. Thomas, M. Am. Soc. C. E., and E. P. Schuleen, Assoc. M. Am. Soc. C. E. (19), to eliminate the element of doubt mentioned previously. The entire model is enclosed in a tank and the air above the model is evacuated so that the ratio of atmospheric pressure to absolute pressure imposed on the model is approximately equal to that of the model scale. By this method cavitation is produced in the model and its form and location become visible to the model operator. Photographs of expected cavitation may thus be obtained and may be supplemented by pressure measurements as well. The practical difficulties of high vacuum testing of hydraulic models are numerous but Professor Thomas has overcome these successfully and has indicated certain advantages of the vacuum-tank method. He has also developed material that will pit under the action of model cavitation and it is possible that, in the future, vacuum-tank testing will show not only the forms of the cavitation but the areas damaged by cavitation as well.

EXPERIENCES OF THE CORPS OF ENGINEERS

By John C. Harrold,[3] Assoc. M. Am. Soc. C. E.

Synopsis

The discovery, in 1935, of severe pitting of the concrete in the conduit entrances of Madden Dam on the Chagres River in the Isthmus of Panama prompted the Corps of Engineers, U. S. Army, to undertake later that year a program of model research on the subject of cavitation. The initial purpose of this research was to develop a design of conduit entrance that would be free from cavitation. The program was later expanded to include a study of cavitation at gate slots in conduits and cavitation around baffle piers in stilling basins of high dams. The major part of this paper is devoted to a discussion of this model research and its effect on the design of structures that come under the jurisdiction of the Corps of Engineers.

The first research was conducted at Carnegie Institute of Technology, at Pittsburgh, Pa., under the direction of Harold A. Thomas, M. Am. Soc. C. E. Later the U. S. Waterways Experiment Station at Vicksburg, Miss., joined in the study. Since 1941 both laboratories have been engaged in cavitation research for the Corps of Engineers. At Carnegie Institute of Technology, models are tested in a vacuum tank in which the air pressure can be reduced to scale and in which actual cavitation can be made to occur. At the U. S. Waterways Experiment Station, ordinary open-air models are used and cavitation pressures are detected with especially designed electric pressure cells capable of measuring rapidly fluctuating pressures in water. These cells are attached to piezometer openings in the face of the model. Both methods have proved highly satisfactory and the results obtained at both laboratories have been correlated.

Introduction

The first high, concrete, flood control dam in the Upper Ohio River Valley—Tygart Dam—was under construction in 1935 when news reached the Pittsburgh District Office of the Corps of Engineers (under whose supervision Tygart Dam was being constructed) that severe pitting of the concrete had occurred in the entrances of the outlet conduits of Madden Dam (19*a*). Since the Tygart conduits were similar in design to the Madden conduits and were to be operated under about the same head, concern was felt lest pitting should also take place in these conduits.

The cause of the pitting in the conduits of Madden Dam appeared to be cavitation induced by the sharp entrance curve in the roof of the conduit. The curve was slightly less sharp in the case of the Tygart conduits, but it was not known whether the difference in curvature would be sufficient to prevent the formation of cavitation.

[3] Senior Hydr. Engr., Office of Chf. of Engrs., War Dept., Washington, D. C.

The possibility of conducting model tests for studying the problem was discussed with Professor Thomas of Carnegie Institute of Technology, who had already conducted model tests of the stilling basin of Tygart Dam for the Pittsburgh District Office. Professor Thomas expressed the opinion that such tests would be feasible and agreed to undertake the tests for the Pittsburgh Office. Model tests of the conduit entrances would first be made in order to determine definitely whether cavitation had been the cause of the pitting at Madden Dam and to provide a general check on the reliability of the methods of testing used. Tests of the Tygart conduits would then be made to determine whether or not cavitation could be expected to occur in these conduits. If so, further tests would be made to determine a remedy for this condition.

These tests showed definitely that cavitation had been the cause of pitting in the Madden Dam conduits. When the results of these tests were called to the attention of the Panama Canal authorities, additional tests of the Madden Dam conduits were authorized with a view to determining a remedy for this condition. Several methods of eliminating cavitation at Madden Dam were tried—reshaping the conduit entrance, constricting the downstream end of the conduit, placing fillers in the stop-log slots, and venting with air. Reshaping of the conduit entrance was finally selected as the most practicable remedy for the condition existing at Madden Dam (19*b*).

A model of the conduit entrances at Tygart Dam was then tested and found to be free from cavitation. Apparently the degree of curvature provided in the entrances of the Tygart conduits was sufficient to prevent cavitation. No change was necessary, therefore, in the design of these entrances. Although the answer was negative in this case, the experience gained from these tests proved to be very useful in subsequent testing of this type.

First Vacuum Apparatus

A special type of apparatus was designed by Professor Thomas and constructed for these tests—the first apparatus for testing hydraulic structures under a vacuum to be built in the United States (19*c*). It consisted of a closed, horizontal-loop, recirculation system in which the pressure could be reduced by an air exhaust pump. Water was recirculated with a centrifugal pump. A part of the system was made large enough to accommodate half-section models of conduit entrances. A glass plate was placed over the conduit on its center line so that the action of the water could be observed.

Conduit Tests

The action at the conduit entrance at Madden Dam in this apparatus has been illustrated by Professor Thomas and E. P. Schuleen, Assoc. M. Am. Soc. C. E. (19*d*). In a similar photograph of the Tygart conduit no cavitation was visible, indicating that the conduit entrance at Tygart Dam will be free of cavitation.

A model of the conduits of another dam was tested in this apparatus in order to study cavitation phenomena in the vicinity of the gate slots. The entrance curve for this design, as a result of experience at Madden Dam, was very liberal. No cavitation was revealed either in the entrance or at the gate

slots under the conditions of maximum head for which this conduit was designed. However, a slight overspeeding of the apparatus (equivalent to increasing the head) gave rise to cavitation flashes at the downstream edges of the downstream gate slot (Fig. 13(*a*)). In order to increase the factor of safety against the occurrence of cavitation in the full-sized structure, a new design of gate slot was developed (Fig. 13(*b*)) which was found to be free from cavitation under this increased head. A slight bevel and rounding was provided on the downstream edges of the gate slot in the improved design. This design has been used in several dams constructed by the Corps of Engineers.

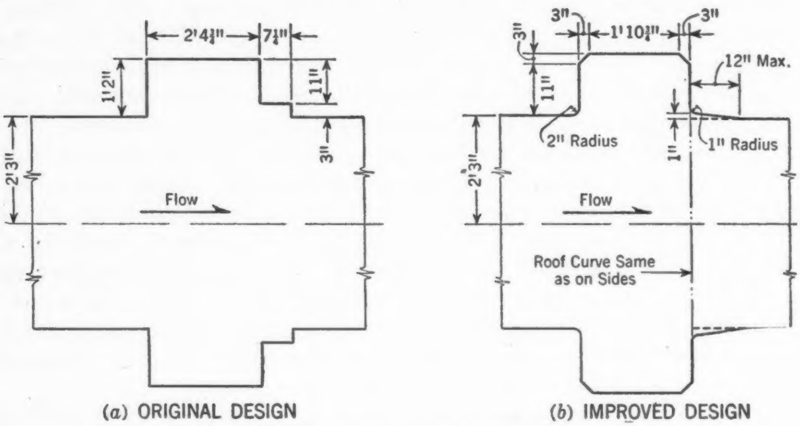


FIG. 13.—IMPROVEMENT IN GATE SLOT DESIGN

BAFFLE PIER PROBLEM

In 1939, four years after the Madden Dam incident, another incident occurred which gave further impetus to the development of the technique of cavitation model testing. A very economical stilling basin design had just been developed in the hydraulic laboratory of Carnegie Institute of Technology for use below the spillway of Bluestone Dam, one of the flood control dams in the Middle Ohio River Valley (Huntington, W. Va., District). This basin consisted of a short horizontal apron at the toe of the spillway with two rows of baffle piers and an end sill on it (see Fig. 14). Fig. 15 shows the water impinging against the baffle piers of this basin. The use of baffle piers enabled a much shallower basin to be adopted than would have been possible otherwise.

After the model study had been completed and the design accepted by the Huntington Office, it occurred to Professor Thomas that there might be cavitation present around these baffle piers. Some well-known foreign engineers who visited the laboratory at that time and saw the model in operation also voiced the same opinion. Professor Thomas, therefore, undertook an analytical study of pressures around cubical blocks in high-velocity streams to determine the likelihood that cavitation would be present. Although this investigation was entirely theoretical and had to be based on certain idealistic assumptions, the results indicated quite definitely that cavitation would be present around

the Bluestone baffles. Not being satisfied with this theoretical analysis, however, Professor Thomas began to give some thought to the development of a

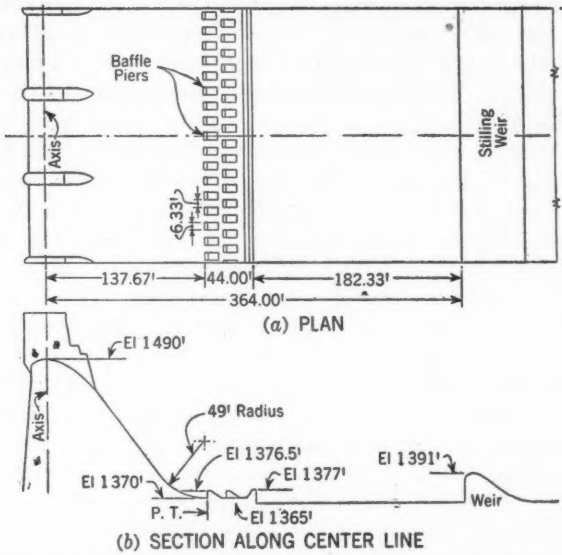


FIG. 14.—SPILLWAY AND STILLING BASIN OF BLUESTONE DAM, IN WEST VIRGINIA; SHOWING THREE BAYS

FIG. 15.—ACTION OF BLUESTONE STILLING BASIN AT MAXIMUM DISCHARGE

vacuum apparatus in which models of stilling basins could be tested. With the help of a research student he was finally able to complete the apparatus shown in Fig. 16. Fig. 16(*b*) is a view of the test chamber.

Improved Vacuum Apparatus

Professor Thomas' improved vacuum cavitation apparatus consists of a closed circulation system, as in the old apparatus; but in many other respects it differs from the old apparatus. The circulation loop is vertical instead of horizontal; and the pump is located two floors below the test chamber instead of on the same level. This latter feature eliminates the principal difficulties experienced with the previous apparatus—cavitation in the pump and air leakage in the pump packing. An ice cooling jacket was also placed on the vertical supply pipe to keep the water at a fairly constant temperature during the tests. Without the cooling jacket, the temperature of the water rises during the tests, making an additional correction in the results necessary. The test chamber is 8 ft long, 2.5 ft high, and 12 in. wide. Air is exhausted by a steam ejector located conveniently near the apparatus, the apparatus being constructed in the steam laboratory of the Institute. Windows were provided in the top and in one side of the test chamber. The top window is removable permitting the placing of models in the chamber. This window is sealed around the edges with modeling clay. Before testing, the water is recirculated for a short time under a high vacuum in order to draw as much dissolved air from the water as possible. Dissolved air has not been found to affect the results of this particular type of testing appreciably but it does produce a cloudy appearance of the water when released, which makes observation of the tests difficult. Dissolved air in large quantities has been found to affect the results of cavitation tests of hydraulic machinery.

Principle of Vacuum Apparatus

In a model study utilizing a vacuum tank apparatus, all features of the model are reproduced to scale, as in the case of an undistorted open-air model, except the atmospheric pressure surrounding the model. It would seem at first thought that the atmospheric pressure, being a unit force, should be reduced in the same proportion as the linear scale of the model. This would be correct, except for the fact that it is desired to simulate cavitation as well as correct pressures in the model. If the properties of water could be changed conveniently so that the water would cavitate, or vaporize, at, say, 1/20th or 1/50th of its natural vapor pressure depending upon the model scale used, it would be satisfactory to reduce the air pressure in direct proportion to the scale. However, it is inconvenient, if not impossible, to make water stay in a liquid state until it reaches such a low pressure. It would vaporize long before reaching this pressure. The temperature of the test water being practically the same as the prototype water, the test water will vaporize at practically the same pressure as the prototype water, not 1/20th or 1/50th of this pressure. A correction, therefore, must be added to the theoretical air pressure (atmospheric pressure divided by the model scale) to make up for the impracticability of changing the vaporizing properties of water.

A numerical example of the method of computing the atmospheric pressure to be used in a model study of this type is illustrated in Fig. 17. The spillway section of a dam has been chosen for the example but the analysis applies equally well to any hydraulic structure having a free water surface.

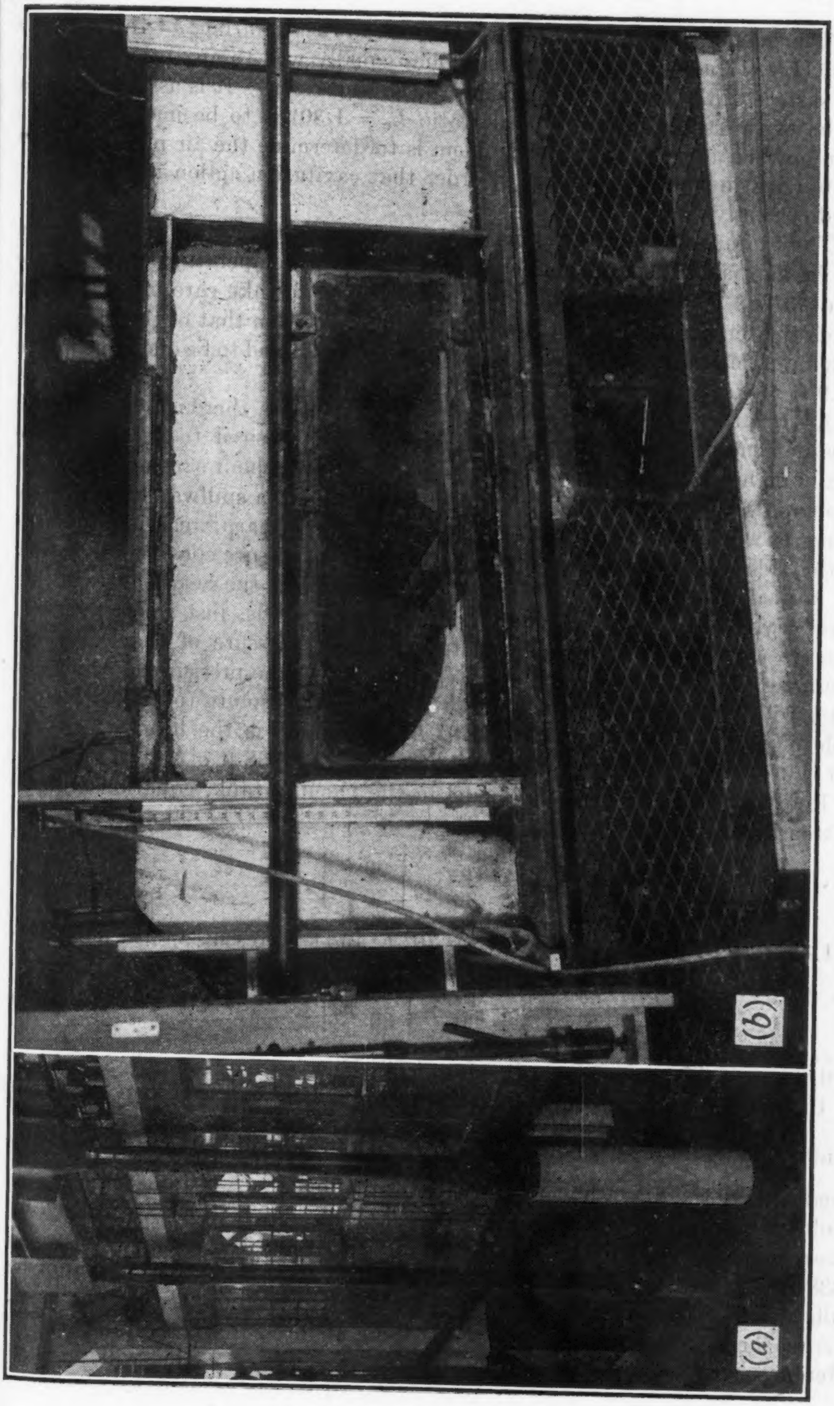


FIG. 16.—IMPROVED VACUUM APPARATUS FOR MODEL TESTS OF STILLING BASINS: (*a*) General View; and (*b*) Test Chamber

The point in question is point B on the downstream face of the spillway crest (Fig. 17(a)); but the analysis applies equally well to any other point on the surface of the structure, including the stilling basin. It is assumed that a model of this spillway (linear scale ratio $L_r = 1/30$) is to be installed in the vacuum tank for testing. The problem is to determine the air pressure to be maintained in the test chamber in order that cavitation action may be simulated correctly.

The example has been simplified by assuming that the water in the model will be kept at a constant temperature equal to the temperature of the prototype water. Later a correction will be developed to take care of the case in which the temperature of the test water is different from that of the prototype water. Atmospheric pressure in the prototype is assumed to be one atmosphere (33.9 ft of water, absolute).

Assume that at point B in the prototype, Fig. 17(a), there is a sharp break in alinement due to faulty construction and it is desired to determine the maximum head, H_p, that can be allowed on the crest without causing cavitation at point B. (The pressure on the downstream side of a spillway crest usually decreases with an increase in head on the crest, and a sharp break in alinement on this side would result in a local lowering of this pressure considerably below the normal pressure for that head.) The head, H_p, is the head that will just cause cavitation to start at point B or, in other words, just cause the local pressure at point B to be reduced to the vapor pressure of the water, p_v. Assuming the temperature of the water to be 70° F, the pressure at which the water will vaporize is $p_b = p_v = 0.838$ ft of water, absolute (obtained from a table of vapor pressures). The head, H_p, therefore, is the head which will just cause the pressure at point B to be reduced to 0.838 ft of water, absolute.

Referring to Fig. 17(b), the model in the vacuum tank, equations can be written for the air pressure, p_{am}, in the tank—

$$p_{am} = \frac{p_{ap}}{30} + \left[\frac{29}{30}\, p_v\right] \quad \text{(2a)}$$

$= 1.13 + [0.81] = 1.94$ ft of water at 70° F; and the pressure, p_{bm} at point B—

$$p_{bm} = \frac{p_v}{30} + \left[\frac{29}{30}\, p_v\right] \quad \text{(2b)}$$

$= 0.028 + [0.81] = 0.838$ ft of water at 70° F.

In Eqs. 2, the first term to the right of the equals sign is the pressure that would exist with the model head, H_m, $\left(= \dfrac{H_p}{30}\right)$, if the air pressure in the tank is reduced by the direct linear scale ratio of the model, 1 : 30, and if the water would stay in a liquid state until a pressure as low as 0.028 ft of water, absolute, is reached. Since water at 70° F will vaporize when the higher pressure of 0.838 ft of water, absolute, is reached, a pressure as low as 0.028 ft cannot be realized. Therefore, to make the water just start to vaporize when the head, h, is reached, it will be necessary to raise the air pressure in the tank by the difference between 0.838 and 0.028, or 0.81 ft of water. This will raise all

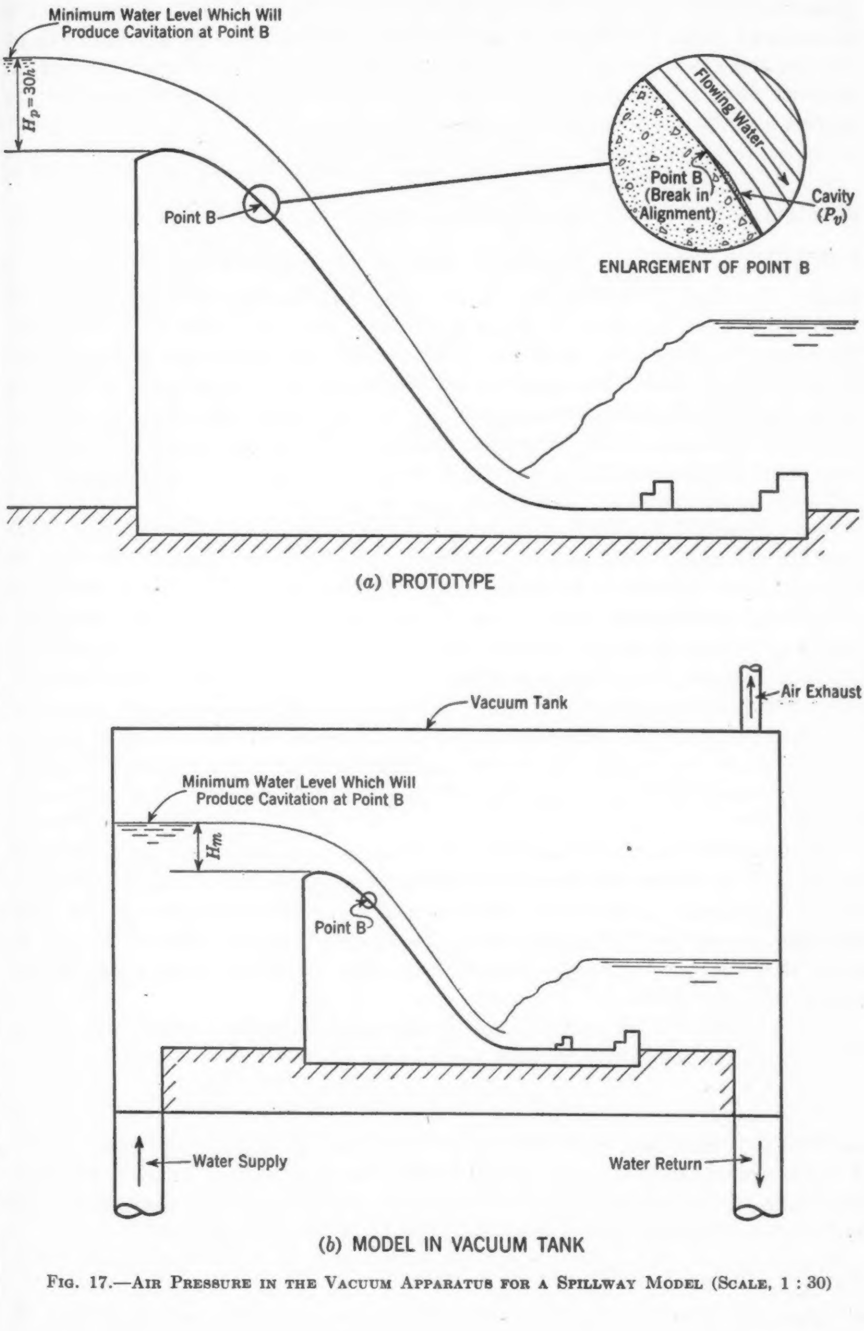


FIG. 17.—AIR PRESSURE IN THE VACUUM APPARATUS FOR A SPILLWAY MODEL (SCALE, 1 : 30)

pressures on the model, including that at point B, by 0.81 ft of water. The pressure at point B will just reach 0.838 ft of water, absolute, when the head, H_m, on the crest is reached. Cavitation, therefore, will just start to form at point B when the head, H_m, is reached, which is the condition required for the correct simulation of cavitation action.

The term 0.81, or $\frac{29}{30}\,p_v$, has been inclosed in brackets in Eqs. 2 in order to distinguish it as the correction factor that must be added to the theoretical air pressure, $\frac{p_{ap}}{30}$, to obtain the correct air pressure p_{am} to be used in the vacuum tank. For the case in hand, it will be noted that the correct air pressure to be used in the tank is 1.94 ft of water, absolute. With this pressure in the tank, the head on the spillway crest can be increased until cavitation is observed to begin at point B, and the head at which this occurs is recorded. When this head H_m is multiplied by the reciprocal of the linear scale ratio, 1 : 30, the result will be the head H_p in the prototype, which will just cause cavitation to start at point B and which, therefore, is the head below which the dam must be operated if cavitation is to be avoided at point B.

Since the air pressure in the tank p_{am} has been increased by 0.81 ft of water over the theoretical, all absolute pressures measured in the tank will be high by this amount. Therefore, all measurements of absolute pressure will have to be reduced by this amount before being converted to prototype values. Measurements of relative pressure (differences in pressure between the air pressure in the tank and the pressure at a given point on the model) will not need to be so corrected, however, because they are independent of the pressure in the tank. The correctness of this latter statement will be seen when it is realized that in ordinary open-air models the surrounding air pressure is considerably higher than the theoretical and yet the relative pressure readings are considered reliable.

It is also desired to call attention to the fact that, in the case of a spillway model such as that in the present example, the phenomenon of separation must also be considered, since end aeration conditions in the prototype may be such that the nappe would separate from the crest before the vapor pressure at point B is reached. It was assumed that this would not take place in the above example.

The equation for the air pressure in the tank, p_{am}, given in Fig. 17(*b*), can be expressed in general terms (21), as follows:

$$p_{am} = L_r\, p_{ap} + (1 - L_r)\, p_v \qquad \text{(3a)}$$

in which p_v is the vapor pressure of both the test water and the prototype water at the specified temperature. If the temperature of the test water is different from that of the prototype water, a review of the foregoing analysis on the basis of this assumption will show that the following formula applies:

$$p_{am} = L_r\, p_{ap} + p_{vm} - L_r\, p_{vp} \qquad \text{(3b)}$$

in which p_{vm} is the vapor pressure of test water; and p_{vp} is the vapor pressure of prototype water. Applying Eq. 3*b* to the foregoing example, it will be seen

that the correction to be added to the theoretical pressure is the difference between the vapor pressure of the test water and 1/30th the vapor pressure of the prototype water.

It will be noted in Eqs. 3 and in the accompanying example that, when the model scale L_r is small, the factor $L_r\, p_{vp}$ is extremely small. In the case of very small models, therefore, this factor may be neglected if great precision in the results is not required and the following approximate formula may be used:

$$p_{am} = L_r\, p_{ap} + p_{vm} \quad \text{(4)}$$

The correction to be added to the theoretical pressure in Eq. 4 is only the vapor pressure of the test water.

Baffle Pier Tests

Trial runs in the vacuum tank apparatus demonstrated that this apparatus would accomplish the results desired. A preliminary model of the stilling basin of Bluestone Dam was installed in the tank and tested under the maximum discharge conditions to which the basin would be subjected. Cavitation showed up immediately around the baffle piers. This matter was then called to the attention of the Corps of Engineers and arrangements were made to have further tests of the Bluestone stilling basin conducted at both Carnegie Institute of Technology and the U. S. Waterways Experiment Station at Vicksburg, with a view to determining the severity of this cavitation condition for the complete range of discharge to which the basin would be subjected and with a view to determining a remedy for this condition, if later considered desirable and feasible. The U. S. Waterways Experiment Station, working with an open-air model, concentrated on determining a streamlined shape of baffle pier that would be relatively free from cavitation and yet be effective as a baffle pier. At Carnegie Institute of Technology tests were continued in the vacuum tank, first on a more exact model of the original basin and then on various shaped baffle piers, including the streamlined shape developed at the Experiment Station. The results obtained at the two laboratories were correlated as the tests progressed.

The tests in the vacuum tank at Carnegie Institute of Technology will be described first. A model of the original apron design with sharp-cornered baffle piers (linear scale ratio $L_r = 1/48$) was first installed in the tank and tested. Fig. 18(*a*) shows the detailed dimensions of these baffle piers. The action of this model in the vacuum tank is shown in Figs. 19(*a*) and 19(*b*). In Fig. 19(*a*), the severity of the cavitation may be judged from the fact that the upstream baffle piers (left hand in photograph) are completely enveloped in cavitation pockets. In Fig. 19(*b*) the relatively mild cavitation is revealed by a smaller pocket visible on the side of the closest upstream baffle.

Inasmuch as these tests and those at the U. S. Waterways Experiment Station (to be described subsequently) indicated that cavitation around the baffle piers of the original design would be quite severe, it was decided to investigate other shapes of baffle piers, keeping the same apron length and end sill, to see if a shape could be developed that would be relatively free from cavitation and at the same time be effective as a baffle pier and structurally rugged. Of

the many shapes tested, the most practicable shape appeared to be the streamlined shape developed by the Waterways Experiment Station and shown in Fig. 18(*b*). The same width of front face, 6.33 ft, was retained in this design, the streamlining being added to the sides.

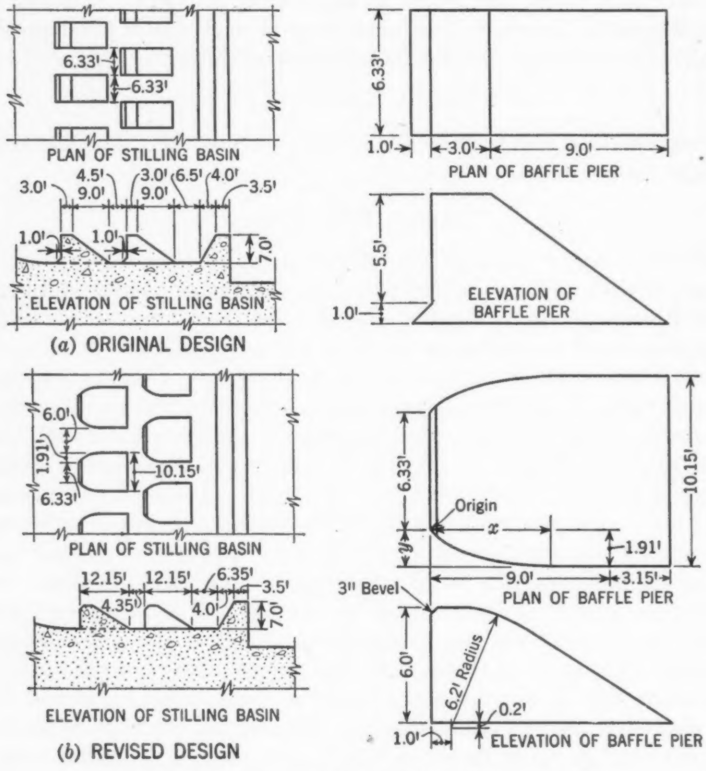


FIG. 18.—DIMENSIONS OF BAFFLE PIERS, BLUESTONE DAM

Figs. 19(*c*) and 19(*d*) show the action of this modified design for the same two discharge conditions as shown in Figs. 19(*a*) and 19(*b*). Cavitation on the upstream row of baffles was completely eliminated by this design. The streamer around the upstream baffle in Fig. 19(*c*) is a cavitation vortex well out in the water away from the face of the baffle pier. There is no cavitation visible in Fig. 19(*d*).

The rounding on the downstream top edges of these baffles is for the purpose of providing better ventilation of the jets of water shooting over the tops of the baffles. This tends to prevent a cavitation streamer from forming on the top of the baffle and causes it to leave the baffle if it does form. The reverse, rounding the upstream top edge, encourages the formation of a cavitation streamer and causes it to cling closer to the top of the baffle. A cavitation streamer that leaves the baffle is assumed to be harmless, whereas one that clings to the baffle is considered harmful.

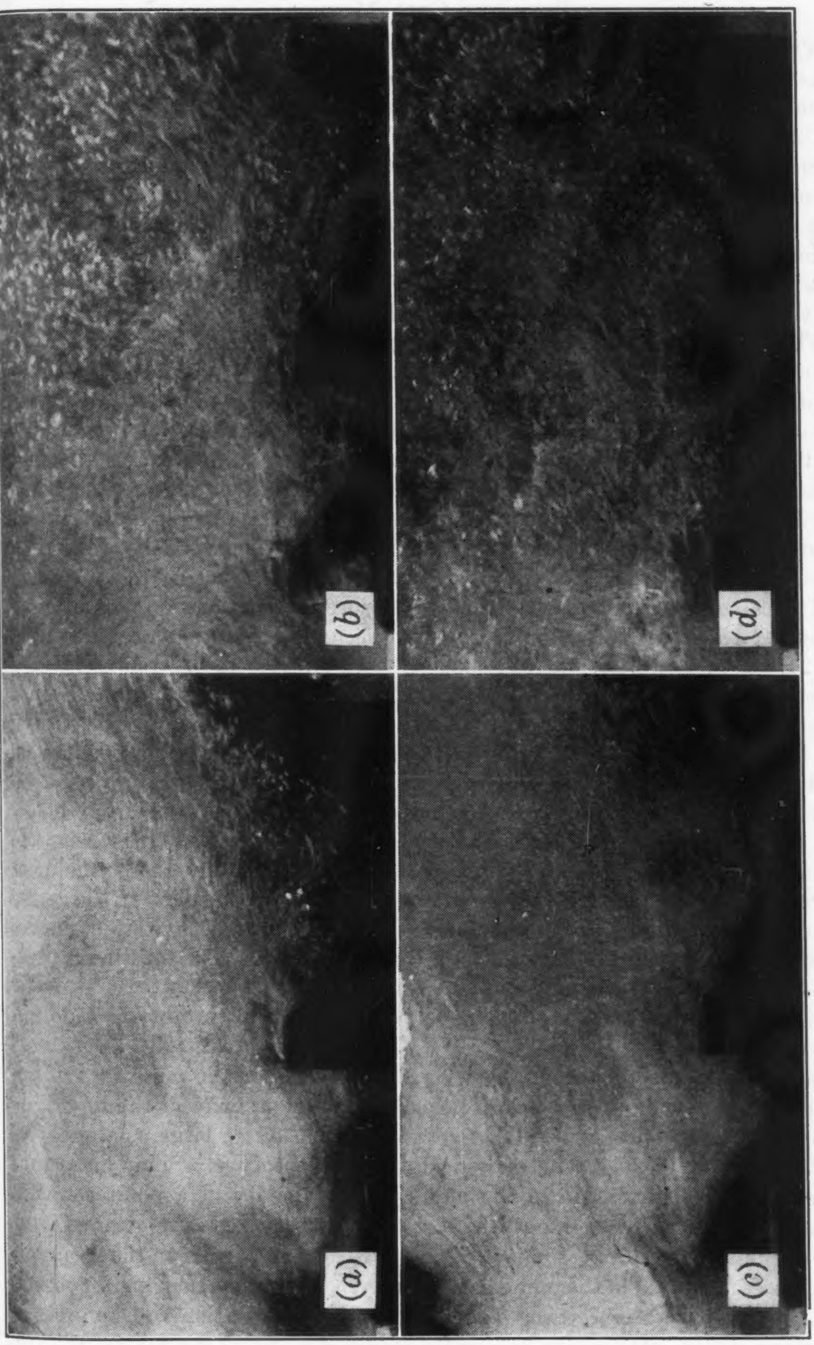


Fig. 19.—Reaction of Baffle-Pier Model, Bluestone Dam, as Tested in Vacuum Apparatus: (a) Maximum Discharge with Original Design; (b) Approximately One Quarter of Maximum Discharge with Original Design; (c) Maximum Discharge with Revised Design; and (d) Approximately One Quarter of Maximum Discharge with Revised Design

Pressure Cell Method

The tests at the U. S. Waterways Experiment Station were conducted on an open-air model for which the linear scale ratio L_r was 1/36. Two baffle piers in each row were constructed with piezometer openings in the faces as shown in Fig. 20(*a*). These piezometer openings were connected by means of copper tubing to a heavy metal pressure chamber in which was fastened an electric pressure cell, as shown in Fig. 20(*b*). (Fig. 20(*b*) shows a connection to a pitot tube but the connection to a piezometer opening is identical.) A cross section of one of the electric pressure cells is shown in Fig. 20(*c*). Several of these pressure cells can be connected to a single amplifier and oscillograph unit, and the pressure at several points recorded simultaneously.

This equipment had been developed in connection with previous tests performed at the U. S. Waterways Experiment Station (20*a*) and had been found very successful in measuring rapidly fluctuating pressures in water. The cell shown in Fig. 20(*c*) utilizes the induction principle. A pressure on the diaphragm causes it to deflect and a change in the air gap causes a change in the current in the front coil. These current variations are amplified and recorded photographically by the oscillograph. Referring to Fig. 20(*b*), it was found that a considerable length of copper tubing could be used without affecting the pressure readings.

Typical oscillograph records of the pressure fluctuations on the original Bluestone baffle piers for the maximum discharge condition are shown in Fig. 21. A dashed line has been superimposed on each record representing a prototype pressure of absolute zero (33.9 ft of water below atmospheric). Pressures below the dashed line are purely imaginary but they have been used as a basis for comparing the severity of cavitation at the various points and for different discharge conditions and baffle shapes. The greater the drop below zero and the greater the percentage of time that the pressure is below zero, the more severe the cavitation is assumed to be. On this basis, the cavitation at piezometer No. 12 (see Fig. 20(*a*)) may be assumed to be the most severe. It will be noted that piezometer No. 12 is on the side of the baffle just downstream from the leading edge, the point where cavitation flashes become visible first in the vacuum tank (see Fig. 19(*b*)). The maximum pressure fluctuation recorded at this point was about 180 ft of water, prototype scale. The pressure was found to be below absolute zero (prototype scale) on the average of 82% of the time.

In interpreting these results, however, it should be kept in mind that in the prototype the pressure cannot drop below the vapor pressure of the liquid, or approximately absolute zero. Therefore, a fluctuation as large as the aforementioned value could not actually occur in the prototype. Some distortion of the pressure distribution in the liquid in the vicinity of a point showing cavitation may also occur due to the fact that cavitation pockets cannot form in an open-air model—a possibility that may reflect some doubt on the accuracy of pressures measured in the immediate vicinity. This is not such an important point, however, because, after it is determined that cavitation will be present in a given design, this design is usually modified until all tendencies toward

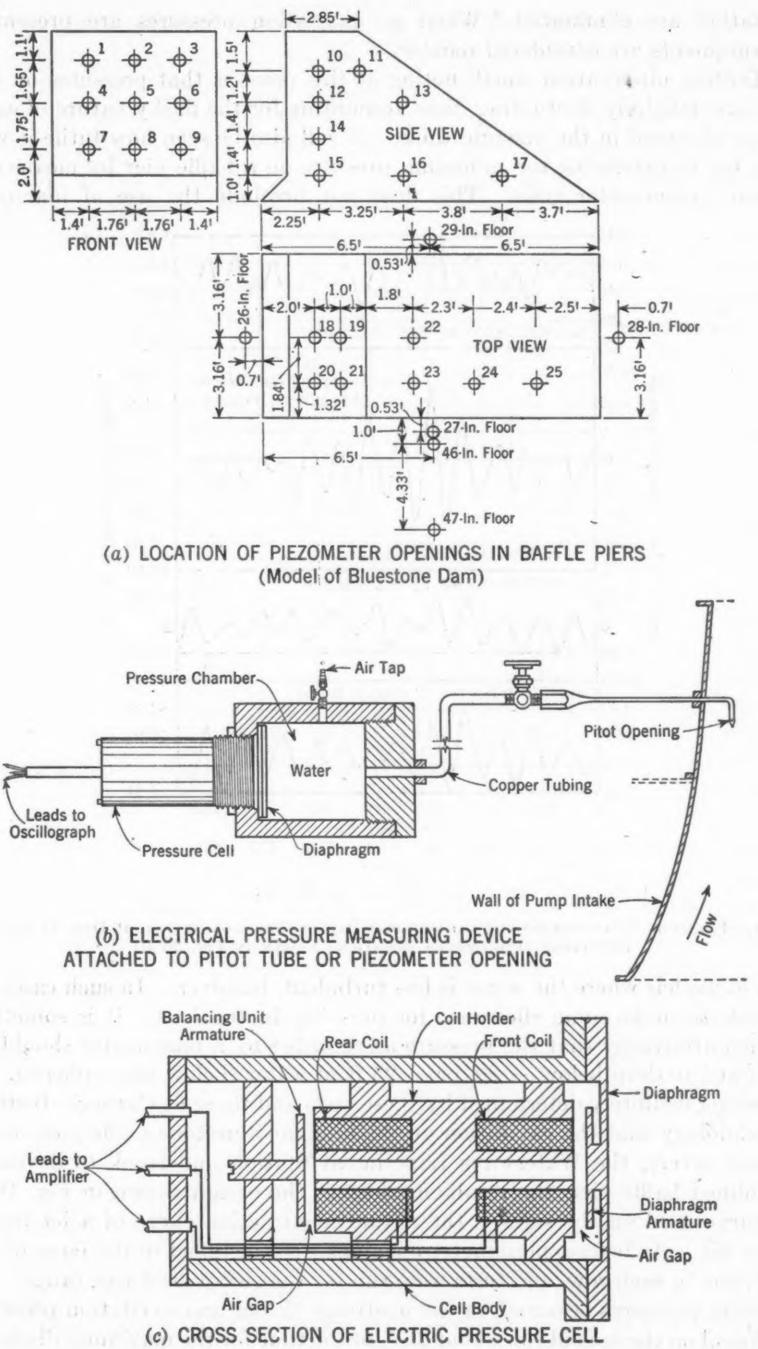


FIG. 20.—ARRANGEMENT OF APPARATUS FOR TESTS WITH ELECTRIC PRESSURE CELLS

cavitation are eliminated. When no cavitation pressures are present, all measurements are considered reliable.

Another observation worth noting at this point is that pressures on baffle piers are definitely fluctuating, thus accounting for the flashy nature of cavitation as observed in the vacuum tank. It will also be seen how futile it would be to try to determine the minimum pressure on a baffle pier by means of an ordinary manometer tube. This does not preclude the use of manometer

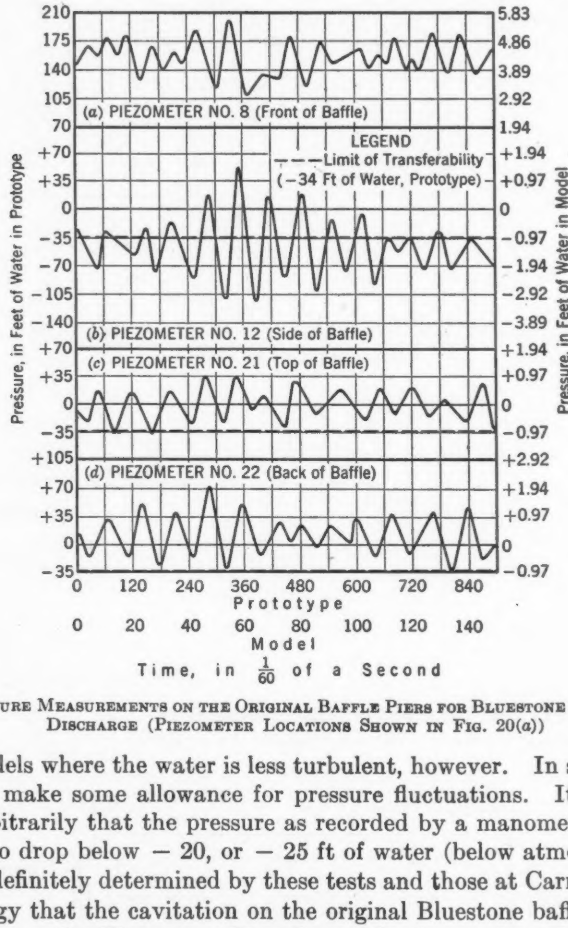


FIG. 21.—PRESSURE MEASUREMENTS ON THE ORIGINAL BAFFLE PIERS FOR BLUESTONE DAM, AT MAXIMUM DISCHARGE (PIEZOMETER LOCATIONS SHOWN IN FIG. 20(a))

tubes in models where the water is less turbulent, however. In such cases it is desirable to make some allowance for pressure fluctuations. It is sometimes assumed arbitrarily that the pressure as recorded by a manometer should not be allowed to drop below − 20, or − 25 ft of water (below atmospheric).

Having definitely determined by these tests and those at Carnegie Institute of Technology that the cavitation on the original Bluestone baffle piers would be quite severe, the Waterways Experiment Station undertook to develop a streamlined baffle pier, the result of which is the design shown in Fig. 18(*b*). The curve used on the sides is the natural contraction curve of a jet from a narrow slit. As before, piezometer openings were installed in the faces of two baffle piers in each row, and tests were run for the entire discharge range. No cavitation pressures occurred on the upstream baffles and cavitation pressures were found on the second row about 25% of the time for the maximum discharge condition.

Pitting of Baffle Piers of Private Power Dam

In connection with the design of the Bluestone stilling basin, the Huntington District Office made a survey of existing dams with baffle piers. Claytor Dam, a private power dam in the Huntington District, was observed to have a dentated end sill, the sides of the dentates of which had been slightly pitted. This dam had just been constructed and was soon thereafter subjected to a large spillway discharge lasting about four days. It was thought that this experience would provide an excellent opportunity to make a model-prototype study of cavitaton which would greatly aid in an interpretation of the foregoing model study results. If the percentage of time that the pressure on the sides of these dentates or baffles was below the cavitation pressure during this discharge could be determined, some idea of the significance of the 25% of time determined for the streamlined Bluestone baffles would be obtained.

Models of the stilling basin at Claytor Dam were tested at both laboratories, and cavitation was observed on the sides of the baffles during the maximum discharge that prevailed during the four-day period. Tests at the Waterways Experiment Station showed that the pressure on the sides of these baffles was below absolute zero (prototype scale) about 25% of the time at this maximum discharge, agreeing almost exactly with the degree of cavitation found on the downstream row of streamlined Bluestone baffles.

As a result of these tests on Claytor Dam, it was concluded that pitting on the downstream row of baffles at Bluestone Dam would not be excessive, particularly in view of the expected infrequency of operation of the Bluestone spillway. The streamlined baffle design shown in Fig. 18(*b*), therefore, was considered satisfactory, provided that these baffles would produce the required energy dissipation. Tests were undertaken to determine the relative effectiveness of the original square-cornered baffle piers and the newly developed streamlined design.

The two lower curves in Fig. 22 show the effectiveness of the original square-cornered baffles. At the maximum discharge, a satisfactory jump can be produced with a 12-ft lower tailwater elevation than would be required to produce a jump on a level apron without baffle piers. A curve of tailwater required to produce an equally satisfactory jump was not determined for the streamlined baffles but, at the maximum discharge, it was found that a 2-ft higher tailwater level was required. Therefore, the streamlined baffles may be considered equivalent to 10 ft of tailwater depth at the maximum discharge which is still quite effective. This 2-ft additional tailwater depth was provided in the prototype by raising the crest of the stilling weir 2 ft above that provided in the original design (see Fig. 14).

Comparison of Two Methods of Testing

The two upper curves in Fig. 22 show a comparison between the results of tests made in the vacuum tank and those made with the pressure meter. These curves represent the tailwater levels required to eliminate cavitation on the upstream row of the original Bluestone baffles. The tailwater levels were raised in each case until, in the vacuum tank, cavitation flashes disappeared

and, in the open-air model, the pressure ceased to fall below absolute zero. These levels were recorded and plotted in the form of curves as shown in Fig. 22. The agreement was remarkably close, but such agreement was not obtained until some of the tests were rerun, as it was difficult to determine the exact point in the vacuum tank at which the cavitation flashes disappeared.

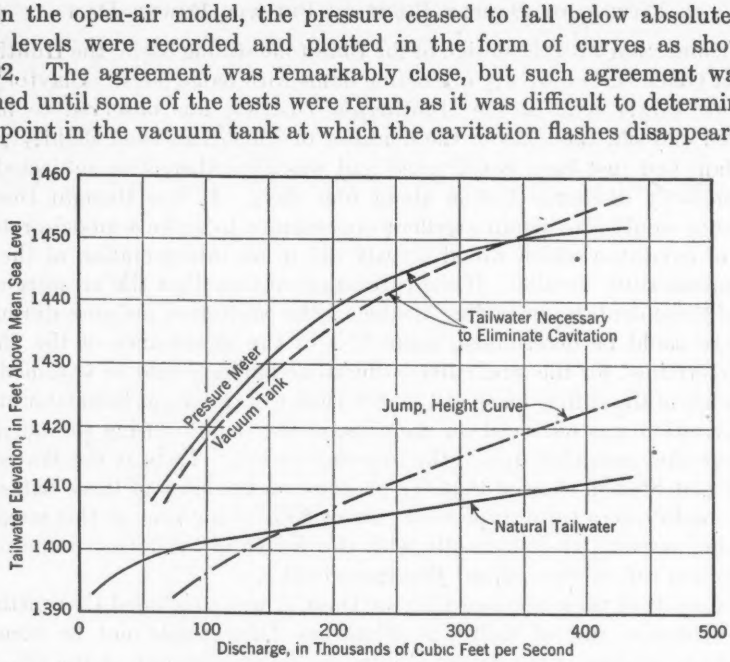


FIG. 22.—COMPARISON OF RESULTS OBTAINED BY VACUUM TANK AND PRESSURE-METER METHODS OF CAVITATION TESTING; ORIGINAL BAFFLE PIERS, BLUESTONE DAM

As a result of these tests on Bluestone Dam, it was concluded that satisfactory results can be obtained by either method of testing—the vacuum-tank or pressure-meter method. Each method, however, has some advantages over the other. The main advantage of the vacuum-tank method is that the cavitation can be seen and its extent observed. The main advantage of the pressure-meter method is that it provides a quantitative measure, permitting a comparison between different degrees of cavitation and enabling the cavitation limit to be determined more accurately and with greater ease. In the pressure-meter method, however, it is necessary to know in advance of the tests where cavitation is likely to occur in order to locate the piezometer openings properly. Also, as previously stated, there is some question in the pressure-meter method as to the effect of the failure to reproduce cavitation pockets on the pressure distribution in the surrounding liquid. The two methods can be used together very nicely, however, and this is recommended whenever possible.

Additional comments on these two methods of testing have been presented elsewhere by J. B. Tiffany, Jr., M. Am. Soc. C. E. (20*a*), and by Professor Thomas and W. J. Hopkins (22).

ACTUAL CASES OF PITTING

Several cases of actual pitting due to cavitation have been discovered in structures built under the direction of the Corps of Engineers, the most notable of which is that at Bonneville Dam on the Columbia River. An inspection of

the spillway of this dam in 1939, two years after completion, revealed considerable erosion of the concrete which was thought to be due in part to cavitation (29a). Since this dam is a run-of-river dam, discharge over the spillway had been practically continuous. The spillway consists of eighteen 50 ft by 50 ft stoney gates on a concrete ogee section about 75 ft high. Water is discharged beneath the gates under a head of about 50 ft and is stilled on a horizontal apron 40 ft below the level of the spillway crest. The horizontal apron is 63 ft long and contains two rows of baffle piers placed at about the third points. The total fall from the upper pool to spillway apron is about 90 ft.

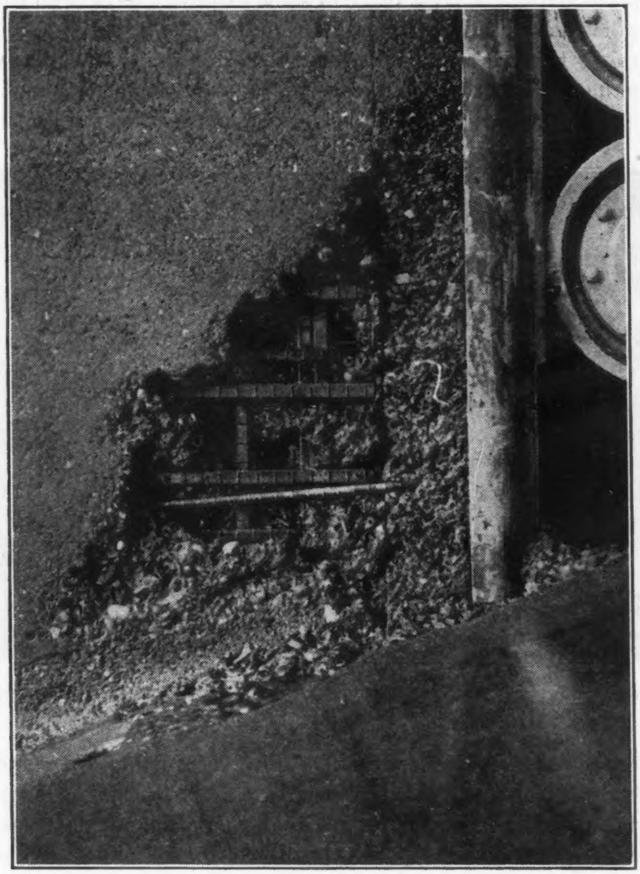


Fig. 23.—Example of Pitting; Downstream from Spillway Gate Slots at Bonneville Dam

Spillway Gate Slots

Fig. 23 shows the erosion that was discovered on the side of a spillway gate pier just downstream from the gate. It will be noted that some erosion has also occurred on the crest. This is typical of all gate piers. It was not known at first whether this erosion was due to abrasion or to cavitation. It was

known, however, that the outside concrete of these piers had not attained its full strength due to cold-weather curing and the use of a pozzolanic admixture. Repair was first attempted, therefore, by refilling some of the holes with good sound gunite made with standard Portland cement, but after another year of operation the holes were found to be in as bad a condition as before. Repair was then attempted by placing $\frac{1}{2}$-in. steel plates over the eroded areas, back-filled with concrete and anchored to the existing reinforcement (29*b*). Plates were placed on the crest as well as on the face of the piers. This work stood up fairly well for a while but in a few months began to fail. The lower halves of the pier plate and of the crest place in some instances have been torn off and the concrete has again eroded in these areas (29*c*).

The cause of this destruction has not been determined definitely. Great pressure fluctuations on the pier face may be responsible for the ripping off of the steel plates. That cavitation is also present is indicated by the appear-

FIG. 24.—VIEW SHOWING PITTING AND EROSION OF BAFFLE PIERS ON THE SPILLWAY APRON OF BONNEVILLE DAM

ance of the eroded areas in the concrete. However, pitting of the steel plates has not occurred except in one instance. In this instance, the pitting could have been due to local irregularities peculiar to this particular plate. As a result of these observations, it would appear that a very mild degree of cavitation is present, one which will not pit steel but which will pit concrete. If further observations prove this to be the case, the final solution to this problem may be to replace the $\frac{1}{2}$-in. plates with heavier plates and to provide better anchorage so that they will not be ripped off. However, if pitting of the plates becomes general, it may be necessary to use a cavitation-resisting alloy steel or to conduct model tests to determine a shape of gate slot that will not give rise to cavitation.

Baffle Piers

The submerged spillway bucket and apron of Bonneville Dam were inspected by a diver. Considerable erosion was discovered on the floor of the apron and bucket and on all faces of the baffle piers (29*d*). The surface concrete of this part of the dam was also known to be weak due to cold-weather

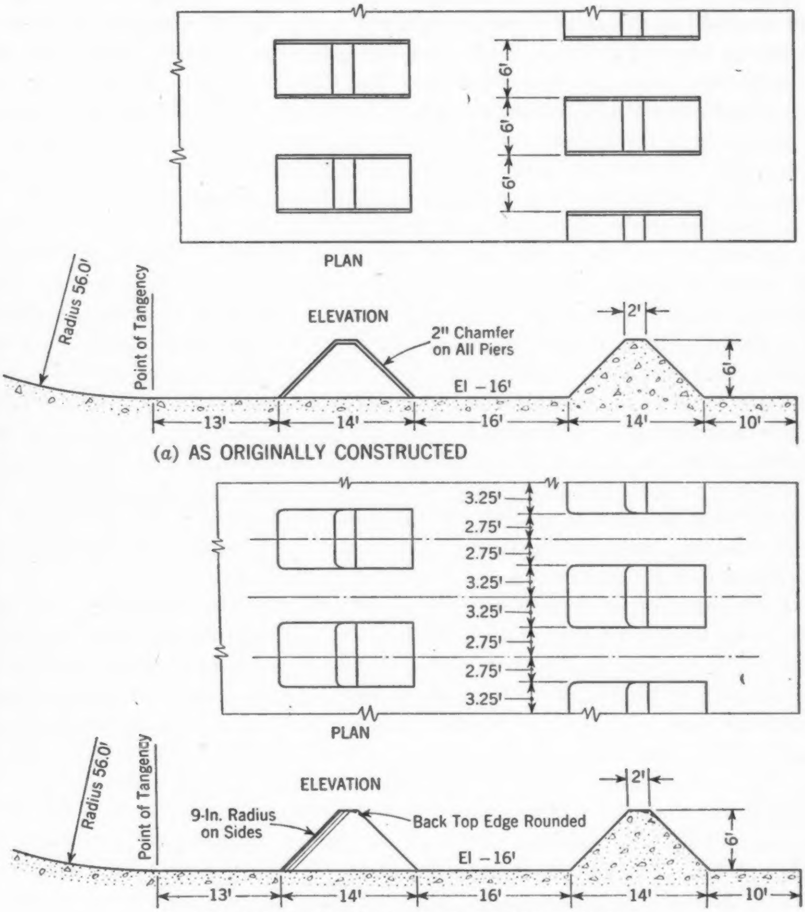


Fig. 25.—Dimensions of Baffle Piers, Spillway Apron, Bonneville Dam

curing conditions. The baffle piers, however, in addition to appearing worn all over, were deeply cut on the sides just downstream from the leading edges, indicating that cavitation may have also been a factor in this erosion. Fig. 24 shows two upstream baffles in a special caisson which was later constructed to repair some of the baffles. Fig. 25(*a*) shows the layout of these baffles on the spillway apron.

Further Model Tests of Baffle Piers

In order that these baffle piers might be properly repaired, it was thought advisable to conduct model tests in the vacuum tank at Carnegie Institute of Technology for the purpose of determining whether or not cavitation had contributed to the erosion of the baffle piers. These tests were undertaken in 1941. A model of the present spillway apron (L_r = 1/48) was constructed and installed in the same vacuum-tank apparatus as was used for the tests on Bluestone Dam (Fig. 16). Tests were run using the operating conditions that actually had been experienced during the first two years of operation and which had caused most of the erosion. A mild degree of cavitation appeared on the sides of the baffles just downstream from the leading edges where the deep holes had scoured in the prototype, under a few of the discharge conditions tested, indicating that cavitation might have contributed to this erosion.

In view of the mild degree of cavitation observed, it was decided to try a simple rounding of the upstream side edges of the baffles to determine whether this would eliminate the cavitation. The width of the baffle piers was also increased from 6 ft to 6.5 ft to make up for the reduction in efficiency caused by rounding the upstream edges and in order to facilitate repairs since the greater thickness would permit a thicker covering of good concrete. Radii of 6 in., 9 in., and 12 in. were studied on the upstream edges. The 9-in. and 12-in. radii eliminated all tendencies toward cavitation and the 9-in. radius was selected for use in repair. The modified design is shown in Fig. 25(*b*). It will be noted that the downstream top edge has also been rounded, as in the case of the Bluestone baffles, in order to provide better ventilation for the jet of water shooting over the top edge of baffle, thus reducing the tendency for cavitation to form on the top.

Tests were also conducted on an open-air model of this spillway (L_r = 1/40) in order to determine the relative efficiency of the original and modified baffle piers. The original baffles were tested first and the tailwater was lowered to the point where the hydraulic jump tended to leave the apron. These tailwater elevations were recorded and plotted in curve form as shown in Fig. 26 (solid lines). The tests were then repeated with the modified baffles and the elevations recorded (dashed lines in Fig. 26). A comparison of these curves will demonstrate that the baffles have about the same efficiency with a 6-ft gate opening and that the modified baffles are slightly more efficient than the original baffles at the 12-ft and 20-ft gate openings. The lower the curve the more efficient the baffle becomes since a jump will take place with a lower tailwater level. Repair of these baffles in accordance with the modified design, therefore, will not affect adversely the efficiency of the stilling basin.

Pitting Simulated in Vacuum-Tank Model

An attempt was made, in connection with the Bonneville tests, to simulate pitting in the model. In view of the mild nature of the cavitation that occurs in the vacuum tank on these small models, and particularly in view of the mild nature of cavitation around the Bonneville baffle piers, considerable difficulty was encountered in finding a material sufficiently weak to pit and yet strong

enough to resist abrasion by the high-velocity water. Several materials were tried consisting of various mixes of cement, sand, and plaster of Paris or Opalite. It was found that a weak material with a hard outside crust was the most successful. The hard crust resisted abrasion but would fail under the pounding of cavitation. Being weak underneath, the material would then pit and form cavities such as those that occur in the prototype. In a test of one

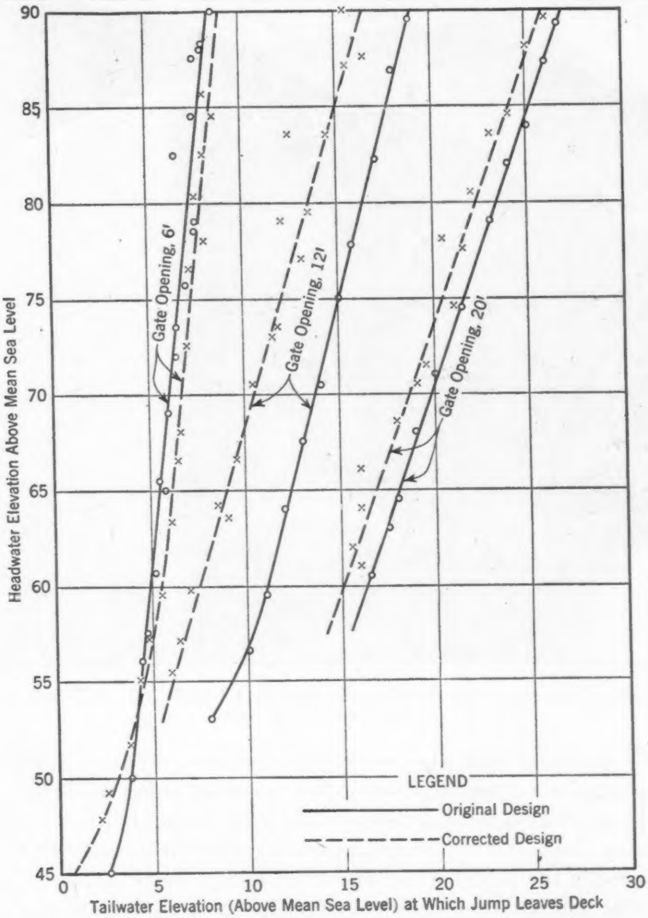


FIG. 26.—RELATIVE EFFECTIVENESS OF THE BAFFLE PIERS IN FIG. 25, AS DETERMINED BY MODEL TESTS

of the original square-cornered baffles, the model had to be "overspeeded," equivalent to subjecting the baffle pier to higher velocities than would exist in the prototype, in order to obtain any appreciable pitting. Some pitting occurred on the sides of the baffle just downstream from the leading edge, where pitting occurred in the prototype (see Fig. 24). These pitting tests were not entirely satisfactory, however, and further research on this subject will be necessary before any reasonable degree of success is possible with this small a model.

Another Example of Damage to Baffle Piers

Fig. 27 shows the effect of high-velocity flow on the baffle piers (height, 9 ft) in the stilling basin of Gatun Dam in the Canal Zone. The steel casting remaining in place is on the upstream face of the pier and is 3 in. thick. The side casting, which has been ripped off by the flow, was 2 in. thick. The front

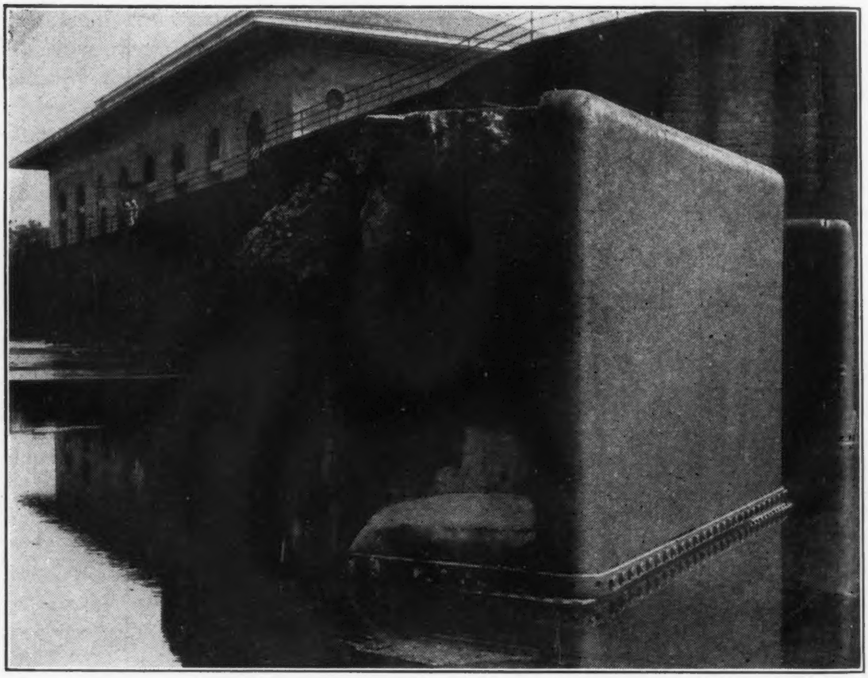


Fig. 27.—Face Plate Ripped from the Side of a Baffle Pier in the Stilling Basin of Gatun Dam

casting was installed with the original baffle piers. The side castings were added later after considerable erosion had occurred along the sides. The side plates were anchored into the concrete with 12-in. anchor bolts. The side plates were probably ripped off by the great pressure fluctuations on the sides such as were measured in the model of the Bluestone baffles (see Fig. 21). The original erosion of the concrete on the sides was probably caused by cavitation.

Proper Use of Baffle Piers

Certain conclusions as to the proper location of baffle piers in stilling basins of high dams can be drawn from the data presented in this paper. Square-cornered, round-cornered, and streamlined baffles were tested. It is obvious from these tests that the greater the streamlining, the higher will be the velocity in which they can be used without causing cavitation. Also, since submergence produces a positive pressure tending to neutralize the negative pressure caused by high velocities, the depth of water over the baffle piers has an important

bearing on the velocity which they will stand. Since velocities are higher and depths shallower in the upstream end of a hydraulic jump basin, only streamlined baffles should be used in this location. Square-cornered baffles should be used well downstream on the apron where velocities are lower and submergence greater. The round-cornered baffles are an intermediate type. Research on this subject has not been extended far enough yet to enable one to state just what velocities and submergences each type of baffle will stand, but it is hoped that future testing will make this possible. The data presented in this paper are not intended in any way to discourage the use of baffle piers but rather to reveal some facts as to their proper use, because the use of baffle piers often results in a more economical stilling basin design and more stable hydraulic jump action.

Pitting in Shaft and Lock Culvert

Two other instances of pitting of concrete in structures built by the Corps of Engineers are of interest. Fig. 28 is a view (looking upward) of the erosion

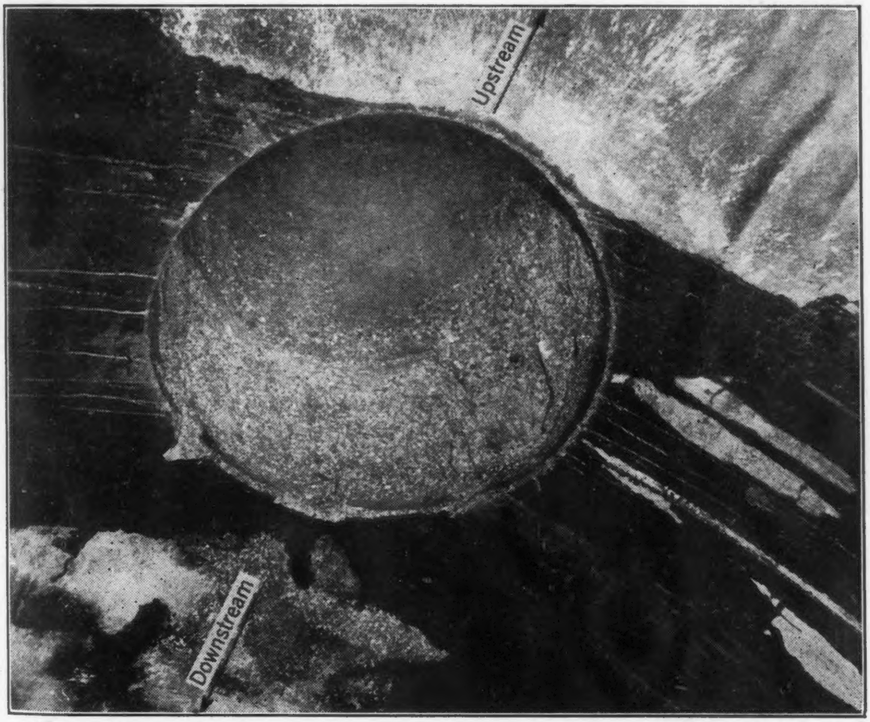


Fig. 28.—Pitting of Concrete Lining at the Bottom of a Vertical Shaft (Diameter, 7 Ft) at Mud Mountain Dam

of the concrete lining which occurred at the bottom of a vertical shaft 50 ft high at Mud Mountain Dam, in the State of Washington, through which water flowed in a downward direction for a period of about three months. Computa-

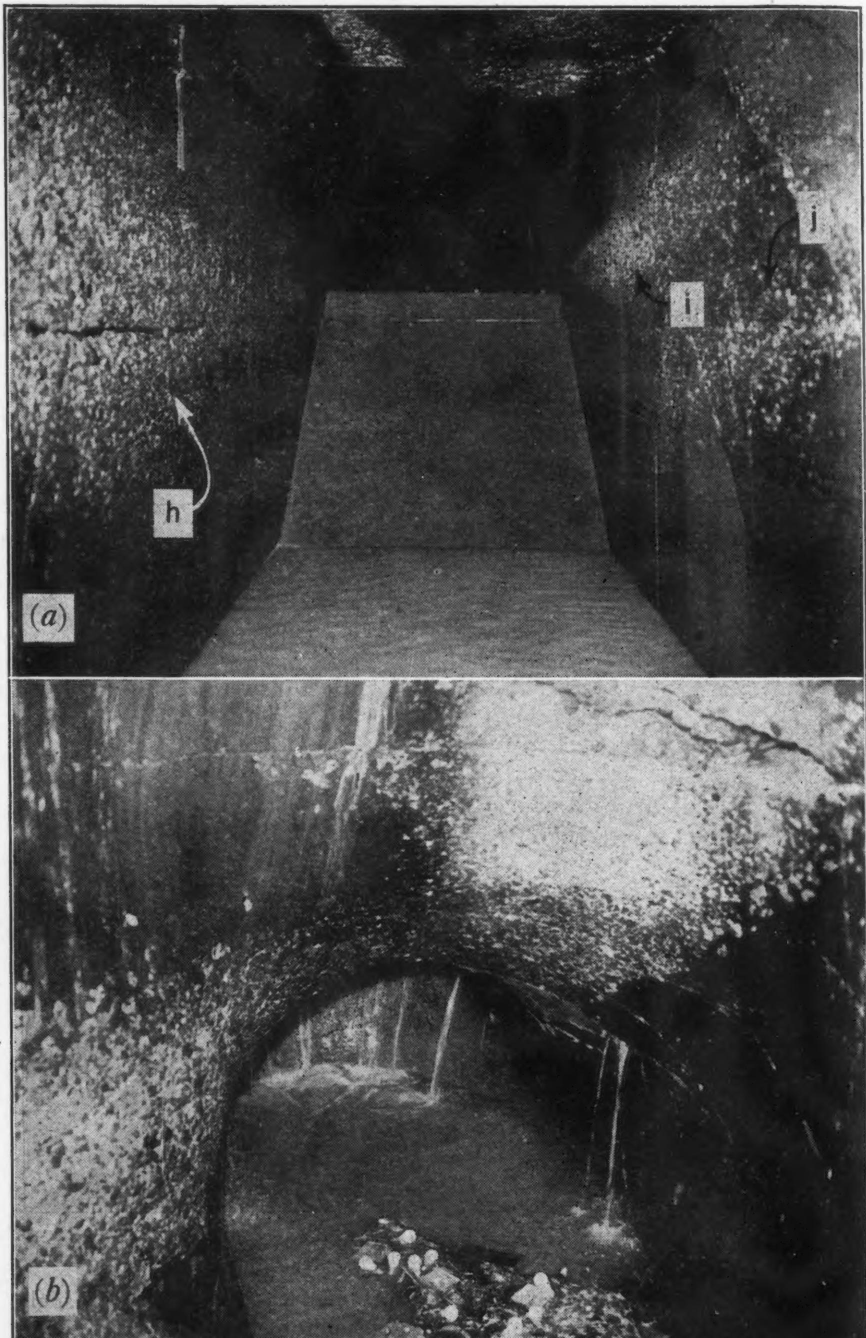


FIG. 29.—EXAMPLES OF PITTING IN THE LOCK-FILLING CULVERT OF WILSON DAM: (*a*) Downstream from a Constriction in the Culvert (7 ft by 9 ft); and (*b*) Downstream from a Sharp Bend

tions showed that cavitation probably occurred at the top of the shaft. Just why the pitting occurred at the bottom is not too clear. Perhaps the vapor pockets did not collapse until they plunged into the pool of water at the bottom of the shaft. The pitting was about 12 in. deep in places.

Fig. 29 shows two views of the pitting that was recently discovered in the filling culvert of Wilson Lock on the Tennessee River. Fig. 29(*a*) shows the pitting that occurred downstream from a constriction in the culvert (note points h, i, and j). The purpose of this constriction is explained elsewhere (23). Fig. 29(*b*) shows the pitting on the inside of a sharp bend in this culvert near its outlet end.

Instructive Lesson

An interesting example of the failure to watch for cavitation in model testing appeared in the technical press in 1940 (24). An ordinary open-air model of the siphon shown in Fig. 30 was constructed and tested. The discharges with

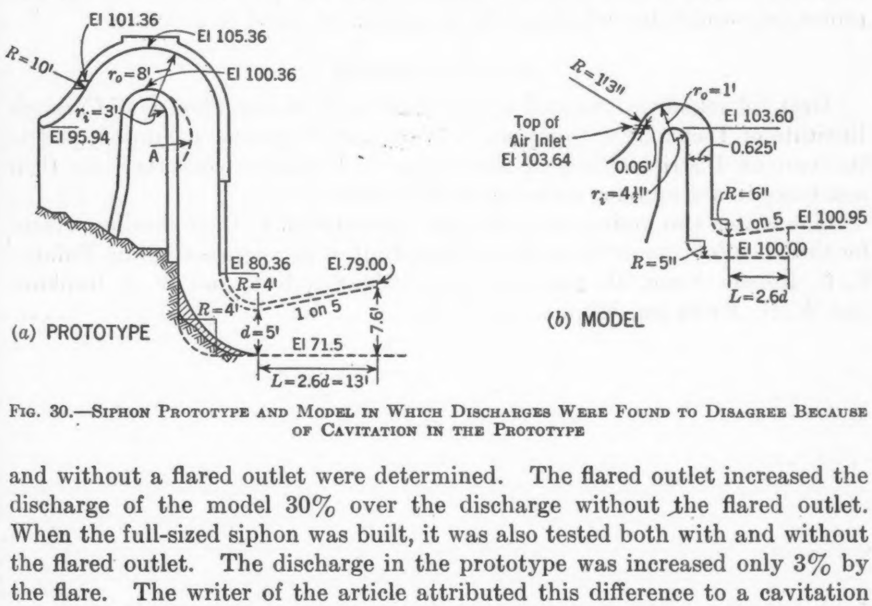


Fig. 30.—Siphon Prototype and Model in Which Discharges Were Found to Disagree Because of Cavitation in the Prototype

and without a flared outlet were determined. The flared outlet increased the discharge of the model 30% over the discharge without the flared outlet. When the full-sized siphon was built, it was also tested both with and without the flared outlet. The discharge in the prototype was increased only 3% by the flare. The writer of the article attributed this difference to a cavitation pocket that formed at point A in the prototype (Fig. 30(*a*)) but not in the model. The lesson is one that all designers should take seriously—to watch for pressures close to the vapor pressure in model studies. If the pressure at any point in an open-air model falls below the vapor pressure (prototype scale), the model is no longer reliable and either the design must be changed to raise all pressures above the vapor pressure, or the model must be tested in a vacuum tank where the action of cavitation can be simulated correctly.

Subjects for Further Research

Cavitation testing has opened a broad field for research. With improved apparatus, it may be possible to test models of entire structures, under a

vacuum, instead of just parts of these structures. Also, having the equipment, it may be possible to develop general laws for the occurrence of cavitation, say around baffle blocks, below gate slots, in curved conduits, siphons, and the like, which will be useful in design. As far as the writer knows there is only one vacuum tank suitable for the testing of hydraulic structures, that at Carnegie Institute of Technology, and only one set of dynamic pressure cells, those at the U. S. Waterways Experiment Station. Perhaps this Symposium will encourage others to undertake useful research along this line.

Certain full-sized tests also appear desirable. It would be interesting to know just how severe cavitation must be in order to damage concrete—that is, how large a cavitation pocket must be or what frequency of collapse is necessary to cause severe pitting. It would also be interesting to know how fast concrete will pit under various degrees of cavitation; also how close the cavitation streamer must be to the structure in order to cause pitting. A general model-prototype study, wherein identical tests are performed in model and prototype, would also help in an interpretation of model test results.

ACKNOWLEDGMENT

Grateful acknowledgment is hereby made to Professor Thomas of Carnegie Institute of Technology, to the U. S. Waterways Experiment Station, and to the various District Offices of the Corps of Engineers concerned for their assistance in assemblying material for this paper.

The writer also desires to express his appreciation to the following persons for their helpful comments on the original draft of this paper: Captain Tiffany, F. R. Brown, Assoc. M. Am. Soc. C. E., Mr. Schuleen, and W. J. Hopkins, and W. H. McAlpine, M. Am. Soc. C. E.

EXPERIENCES OF THE BUREAU OF RECLAMATION

By Jacob E. Warnock,[4] M. Am. Soc. C. E.

Synopsis

With the trend to larger and larger hydraulic structures, cavitation and the resultant pitting has become a major problem to the hydraulic designing engineer. Subatmospheric pressures in smaller structures were of little consequence, but with the increase of head in more recent structures, the approach of subatmospheric pressures to absolute zero as their limit has created previously unheard of situations. Experience in the laboratory and in the field shows that prevention of cavitation is fundamentally a function of design.

Introduction

Pitting due to cavitation is not new to the Bureau of Reclamation, U. S. Department of the Interior. Trouble was experienced as far back as 1910–1920 in outlet works, but recent examples have been more severe in extent due to the increased head. In discussing the arrangement and design of outlet works, J. M. Gaylord and J. L. Savage, Hon. M. Am. Soc. C. E. (25), in 1923 stated that "Most of the difficulties with outlets built by the Bureau of Reclamation can be attributed to the effects of vacuum in the conduits below the regulating devices." However, it was not until recently that pitting has appeared on the surfaces of water passages normally considered to be open channels. Two examples of conduit flow are described herewith—the needle valves at Boulder Dam (Arizona-Nevada) which discharge into the atmosphere, and the balanced valve outlets at Shoshone Dam (Wyoming) which discharge into short conduits. Two examples of pitting in open channels are also shown—in the Arizona spillway tunnel at Boulder Dam and on the spillway pier faces at Parker Dam, both on the Colorado River.

The remedial measures in each case were made possible by laboratory studies. In fact, the occurrence of cavitation and pitting in the large hydraulic structures constructed in recent years would be more prevalent were it not for the availability of hydraulic laboratory facilities. A careful exploration of the pressures within a model can detect those conditions which, when transferred to a prototype structure, will cause the intermittent subatmospheric pressures producing cavitation. At one stage in the design of the spillway for Grand Coulee Dam (26), in Washington, a dentated lip at the downstream end of the apron eliminated the impingement of the high-velocity flow directly on the river bed downstream from the apron, reduced the scouring effect of the turbulent flow, and materially reduced the roughness of the water surface in the stilling pool. Minute examination of the pressures on the faces of the dentates in a

[4] Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

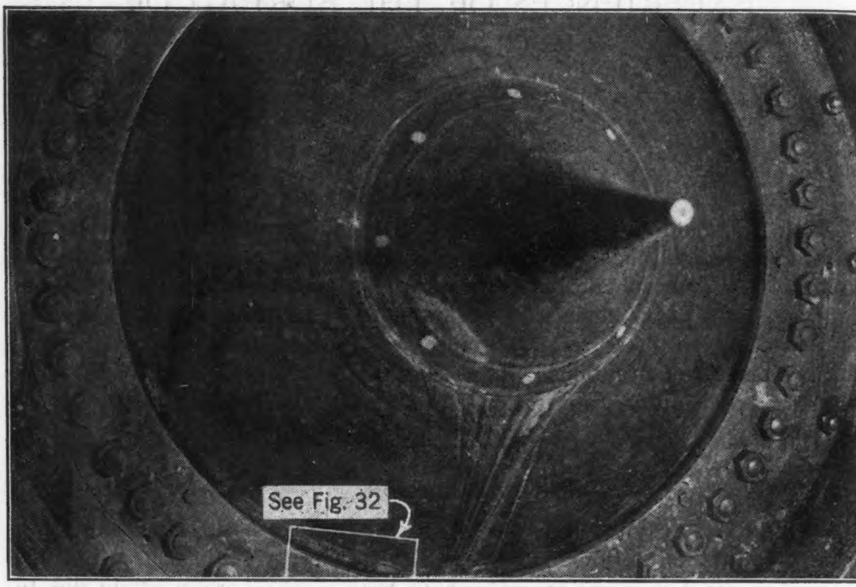


FIG. 31.—PITTING OCCURS AROUND THE PERIPHERY OF A LARGE NEEDLE VALVE, IMMEDIATELY BELOW REGIONS OF SUBATMOSPHERIC WATER PRESSURES

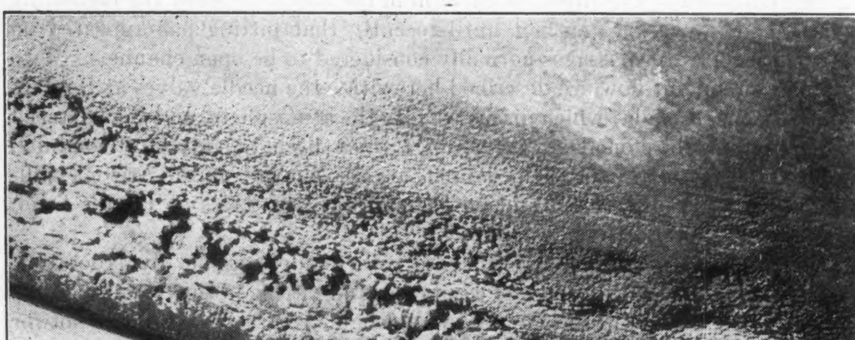


FIG. 32.—ENLARGED VIEW OF SECTION IN FIG. 31 EMPHASIZES THE SEVERE PITTING ON THE SHOULDER OF THE NEEDLE VALVE WHICH REQUIRES EXPENSIVE MAINTENANCE BY WELDING

1 : 40 model and a 1 : 15 model showed that cavitating pressures would occur in the prototype which would have destroyed the piers in a very short time. This condition was the principal factor in continuing the laboratory studies which resulted in the adoption of the bucket type of energy dissipator.

The destruction of the conduit roof in the outlets at Madden Dam, on the Chagres River in the Isthmus of Panama, by pitting from cavitation was the incentive for a complete model study of the outlets (27) for Grand Coulee Dam. Aside from the criterion of efficiency, the emphasis throughout the tests was to prevent pressures from occurring at any point in the conduit which would cause cavitation. One of the early designs of the upper and intermediate outlets showed subatmospheric pressures of such intensity that, had the design been constructed, cavitation would, without doubt, have been so severe as to hamper, if not completely prevent, successful operation. In the original design the fact was overlooked that the frictional forces in the sloping conduit were insufficient to overcome the accelerating force due to gravity, a condition which became readily apparent in the experimental studies.

There was a period prior to the Madden Dam incident when it was difficult to demonstrate that cavitation and pitting could occur in a hydraulic structure in the same manner as it has occurred in hydraulic machinery such as marine propellers, turbines, and pumps. With the experiences at Madden Dam and in certain Bureau structures, augmented by laboratory investigations, the importance of this cavitation problem is fully recognized by Bureau engineers.

The Gaylord-Savage report (25) describes outlet structures and recounts the difficulties experienced in the excessive maintenance due to damage from cavitation. Although the theory of cavitation at that time differed materially from the current conception, the adverse condition was even then associated with extreme subatmospheric pressures. It was realized that the erosion or pitting was an action accompanying subatmospheric pressures, but the cause was not completely understood. At first the pitting was believed to be a direct result of the making and breaking of the vacuum in the immediate vicinity.

As is now the case, one of the most practical remedies applied to the discharge conduits installed in early periods was the introduction of air immediately below the regulating device. Air was admitted to the discharge conduit in a number of instances during the first years of operation, but, in the light of air-requirement tests made in recent years on both model and prototype structures, it is doubtful if the air supply in most cases was either adequate or properly installed. The location of the air inlets in the conduits is often more important than the size. Thus, improper location might have been one of the main factors contributing toward failure of some of the early systems.

Streamlining the needle tips is considered the only practicable means of eliminating damage to this part of the valve; however, damage could be reduced to a minimum by restricting the valve operation to noncritical openings determined by detailed pressure measurements on the prototype or by model tests, or both.

Damage to Needle Valves

Initial operation at Boulder Dam and Alcova Dam, in Wyoming, produced severe erosion of the needle valves in the outlet structures which was expensive to maintain, since the valves were not readily accessible. The type of damage is shown on the needle in Figs. 31 and 32. In this case, the damage was produced in a relatively short time. Detailed records are not available, but the time was probably about one month at the one-half open position with a head of 145 ft.

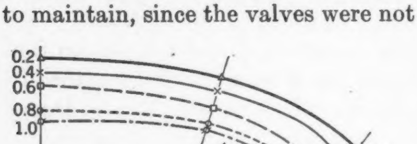


Fig. 33.—Pressure Distribution on the Nozzle of a 72-In. Needle Valve (Head, 516 Ft) Shows Regions of Low Pressure Which Correlate Perfectly with the Occurrence of Pitting

Piezometers installed in the nozzle of one of the 72-in. valves at Boulder Dam showed pressures near absolute zero in a zone (Fig. 33) immediately upstream from the region of erosion or pitting throughout the entire range of the valve. Since the installation of pressure equipment in one of the 72-in. needles would have been intricate, a homologous needle valve with an exit diameter of 5 in. was installed for testing in one of the tunnel-plug outlets at Boulder Dam. The pressure distribution on the needle and nozzle is shown in Fig. 34 for various openings of the valve under a constant head of 150 ft.

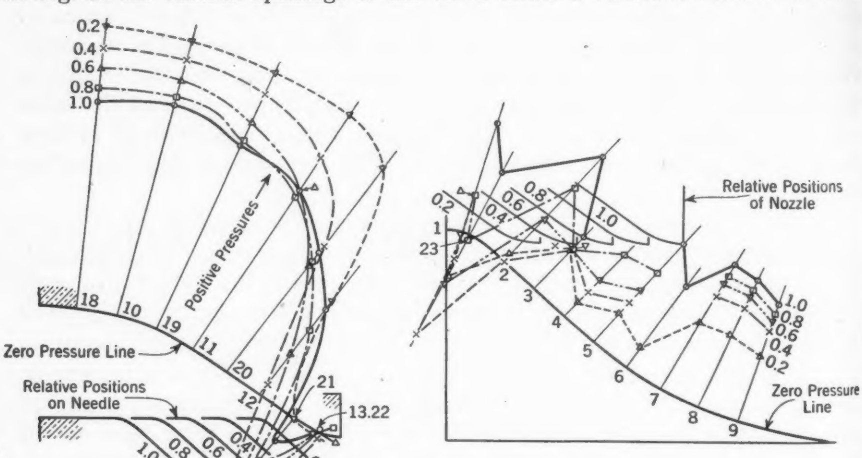


Fig. 34.—Detail Data from Extensive Tests of a 5-In. Scale-Model Needle Valve Further Confirm the Pressure Measurements and Pitting Observed on the Prototype

In general, the pressure conditions were most critical at a valve opening of approximately 40%. The results of a wear test (Fig. 35) are shown after six

days of operation under a total head on the valve of 460 ft with the valve opening of 40%—the severest condition.

A number of designs were studied in the laboratory using the same 5-in. valve and a design was developed which produced positive pressures at all valve openings and at all heads. This design, when subjected to a wear test, under the same conditions as the original design (except at an opening of 20%, the opening at which the severest conditions occurred), showed no sign of pitting on the needle after eighty-four days of operation (Fig. 36).

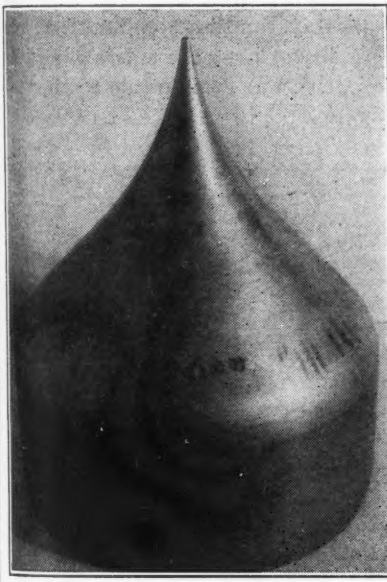


Fig. 35.—Pitting on the Needle of a 5-In. Valve, According to the Original Design, After Six Days of Operation (40% Opening and H = 460 Ft)

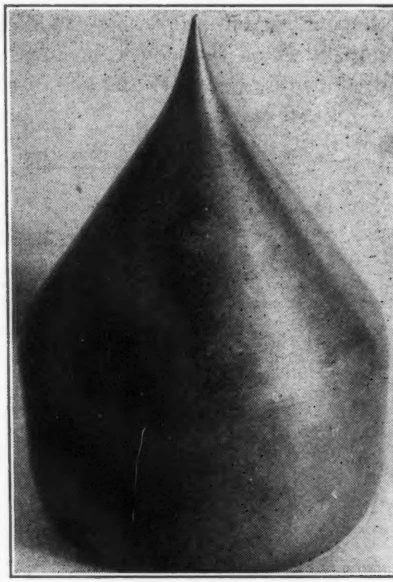


Fig. 36.—Absence of Pitting on the Needle of a 5-In. Valve, According to the Improved Design, after 84 Days of Operation (20% Opening and H = 460 Ft)

The original design of nozzle had an expanding water passage which tended to lower the velocity and cause a regain of velocity head. This yielded a higher discharge but created low-pressure areas in the valve. When forces in the low-pressure areas were of sufficient intensity, they produced cavitation with the accompanying pitting.

From the high-head studies of the 5-in. valve, certain specifications were developed to maintain positive pressures on the needle and nozzle of the valve at all openings. The angle between the needle and the nozzle must not be divergent in the direction of flow. The needle and nozzle profiles may be parallel and still maintain positive pressures, but a convergence of one to three degrees is preferable. The seat must be on the tangent portion of the needle; that is, the base diameter of the needle cone must be slightly larger than the outlet diameter of the nozzle. The valve nozzle should have no point of inflection; it should have a sharp edge to maintain the minimum section at

the outlet of the valve nozzle and should permit free access of air to the jet at the point of emergence.

The high-head studies on the 5-in. valve gave positive proof that the 72-in. valves were pitted by cavitation and that elimination of the severe subatmospheric pressures causing the cavitation was possible by the redesign of the hydraulic passages of the valve. In eliminating the low pressure by using a sharp-edged nozzle in the improved design the discharge capacity was reduced.

Using a 6-in. valve equipped with piezometric taps throughout the profile of the needle and the nozzle, the design was altered further by increasing the outlet and equatorial diameter and the needle travel until a combination was found in which the cavitating pressures were absent over the entire range of valve opening (Fig. 37) and the discharge capacity was comparable to that of the original valve.

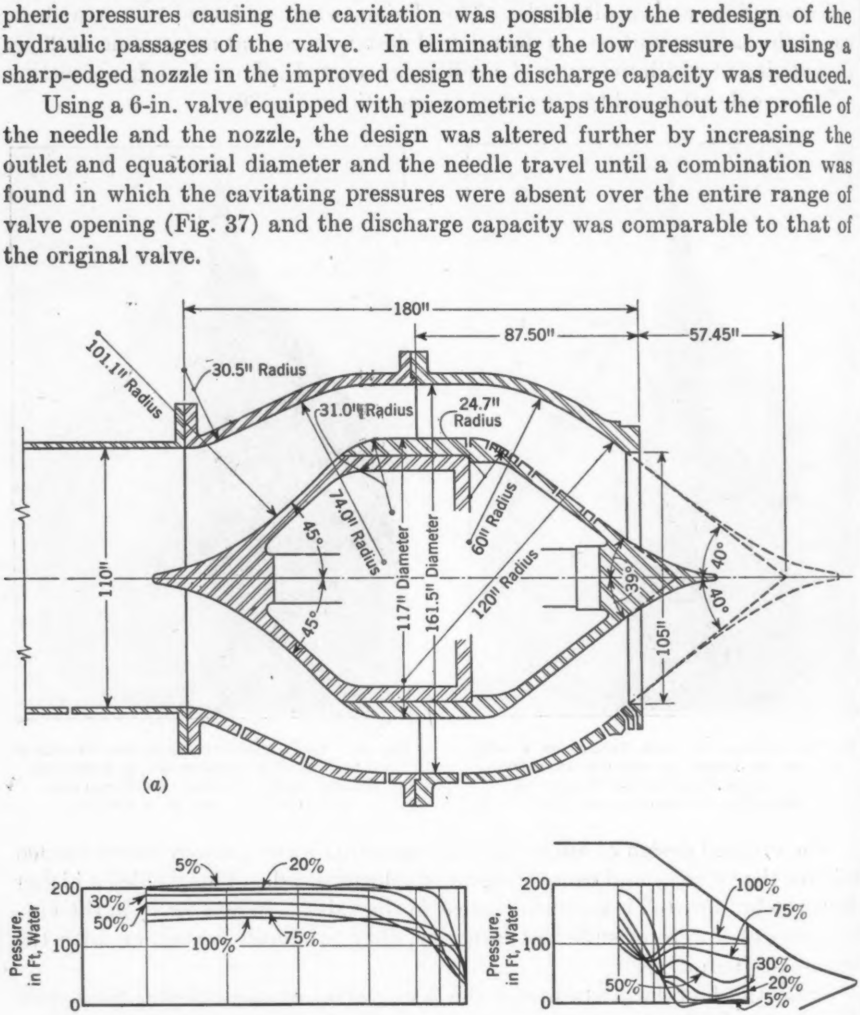


FIG. 37.—COMPLETE ABSENCE OF CAVITATING PRESSURES ON THE NEEDLE AND THE NOZZLE OF A NEW DESIGN OF NEEDLE VALVE, AS DETERMINED FROM A 6-IN. MODEL

Although the new design has not been used in a field structure, the satisfactory operation of the 5-in. valve with positive pressures throughout, under a head of 460 ft for eighty-four days, indicates similar satisfactory operation in a larger valve of the new design. As designed for Friant Dam in California, the

valve in Fig. 37 has an inlet diameter of 110 in. and an outlet diameter of 105 in.

Subsequently, sharp-edged nozzles were installed on certain valves at Boulder Dam and the pressure results show a marked improvement in distribution (Fig. 38). Field reports indicate a minimizing of pitting on those valves so equipped. Operation of the valves with the original nozzle shape at openings where the subatmospheric pressures were less severe, as indicated by model tests, has also tended to reduce the amount of pitting.

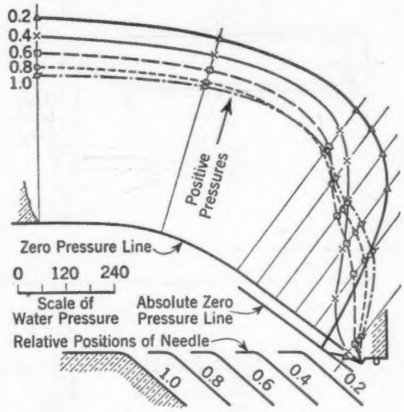


FIG. 38.—PRESSURE DISTRIBUTION ON THE NOZZLE OF THE 72-IN. NEEDLE VALVES AT BOULDER DAM, WITH REVISED PROFILE OF NOZZLE

SHOSHONE DAM BALANCED VALVES

The 58-in. balanced valves in the lower outlet tunnel in the south canyon wall at Shoshone Dam, near Cody, Wyo., were installed in May, 1915. Although this type of valve had already required considerable maintenance in the installations at Roosevelt Dam, in Arizona, and Pathfinder Dam, in Wyoming, it was adopted because of the lack of a better design.

The valves were in operation only a few seasons when it became evident that seasonal maintenance similar to that at the older installations would be required. Pitting of the downstream faces of the valve needles and severe damage to the discharge conduit walls immediately below the valves occurred during extended periods of operation. Patching with various materials or filling the pitted areas by arc-welding with different metals was of no avail. With few exceptions the patches eroded more rapidly than the parent metal.

In 1930–1931, an attempt was made to relieve the Shoshone situation by installing twenty-four 2-in. pipes and an 8-in. air duct below each valve (Fig. 39(*a*)). A marked increase in the intensity of the noise accompanying the discharging water resulted, and the experiment was considered unsuccessful. Because of the failure of the vent system, resort was made to the original method of maintenance and the valves were used as little as possible. The pitting was serious and the repairs inadequate, but a more practical method of repair was not apparent.

During the season of 1942, the valves at Shoshone were operated at almost full capacity over an extended period in order to regulate flood flow and prevent crop damage downstream. Damage to the outlet structure was severe and maintenance measures became critical.

The concrete for several feet downstream from the metal lining in each conduit had been eroded severely and most of the twenty-four 2-in. pipes embedded in the conduit during the 1931 revision had been torn out in the eroded area (Fig. 40(*a*)). The "semi-steel" (high-test gray iron) conduit liner

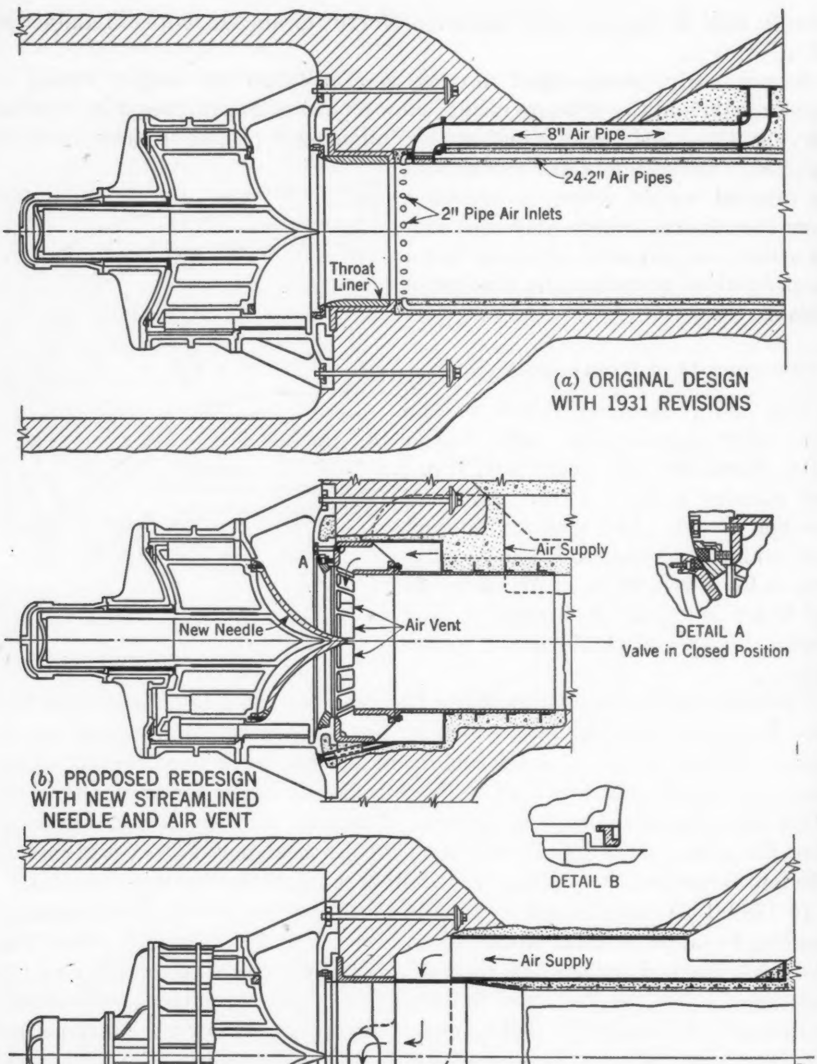


FIG. 39.—BALANCED VALVES IN THE LOWER OUTLETS AT SHOSHONE DAM

below the valve was pitted severely and the face of the needle (Fig. 40(*b*)) had badly pitted areas (by operation in previous years) on which several kinds of

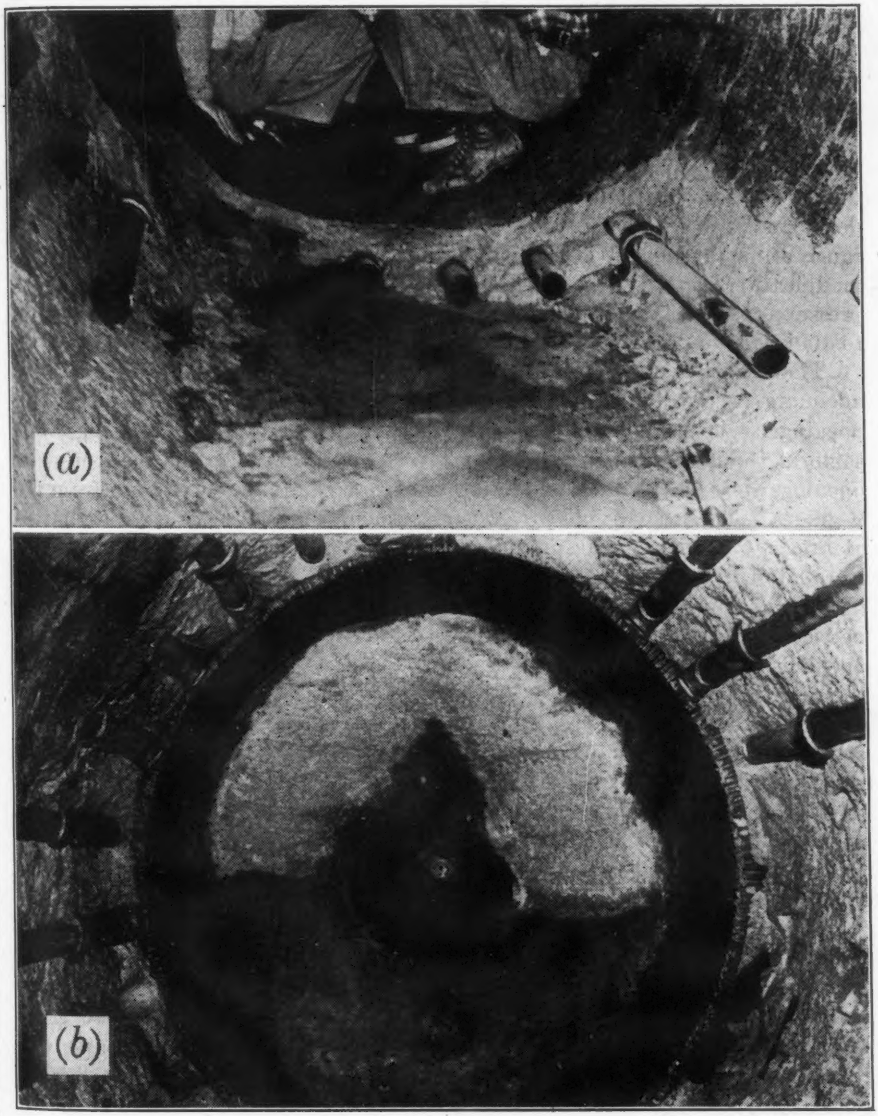


FIG. 40.—REMAINS OF 2-IN. AIR PIPES, SHOSHONE DAM: (*a*) Vent of a 52-In. Conduit Facing Downstream from the East Valve; (*b*) East Conduit

metal had been tried—none satisfactorily. The areas of greatest pitting were below and above the valve guides, where only 3/4 in. of the original 2 in. of parent metal remained. The extensive welding of previous years, on the

needle face, is apparent in Fig. 40(*b*). The end of the metal liner was cut off in 1931 when the present installation was made.

Hydraulic laboratory model studies were made to evolve means of minimizing or eliminating the severe damage, to reduce the unreasonably high seasonal maintenance, and to remove the danger of a possible failure of the water-release system. This problem involved an extensive study of the pressure distribution in the valves and the discharge conduits.

Three alternatives were developed: (*a*) The range of valve opening was determined in which damage would be minimized until materials, unobtainable due to wartime restrictions, become available; (*b*) a redesign (Fig. 39(*b*)) was developed in which adverse pressure conditions were eliminated over the entire range of operation, but at some sacrifice in the discharge capacity; and (*c*) a modification of the present installation (Fig. 39(*c*)) was developed in which pressure conditions were acceptable over a range of valve opening from 25% to 100% with no reduction in the discharge capacity.

The model tests showed that the present prototype vent system is inadequate to prevent cavitation for all except a very small range of valve openings. Insufficient air is supplied between 23% and 70% openings, and some of the 2-in. vent pipes on the invert and crown become ineffective at openings above 85%, due to eddies forming immediately downstream from the V-guides. These conditions precluded safe operation of the present installation at ranges of valve opening other than 70% to 85%. Studies of the present installation indicated that the pitting on the valve needles was most severe between openings of 14% and 25%, and that damage to the conduits resulted between 23% and 70% valve opening. The damage to the conduit at these openings probably rendered the air-supply system ineffective and aggravated the destructive action for larger valve openings.

Damage by cavitation and pitting on the valve needles and discharge conduits can be eliminated entirely by a major revision (Fig. 39(*b*)) of the needle tip, the valve seat, the conduit throat, and the aeration system. This solution will reduce maintenance costs to a minimum and the valves can be operated at any opening without fear of damage due to subatmospheric pressures; but it will reduce the discharge capacity by approximately 20%, a factor to be considered in future revisions.

Minor alterations of the present structure (Fig. 39(*c*)) will involve: (*a*) Streamlining of the sealing edge of the plunger; (*b*) removal of a part of the bronze sealing ring by chipping and grinding; (*c*) removal of the throat liner; and (*d*) revamping of the air-supply system. Aeration equivalent to three 12-in. ducts would be adequate in this arrangement, but slightly more area was recommended as information on air requirements in high-velocity flow is limited. Operation of this modified design at openings smaller than 23% will have to be avoided to prevent damage to the needle. The discharge capacity is not affected noticeably by the modification.

Since materials have been unobtainable to make either the minor alterations or the major revision, the valves were operated during the 1943 irrigation season in the valve-opening range at which the subatmospheric pressures were the least severe. After thirty-five days of operation at 75% opening, the valve

itself showed no evidence of additional pitting, and a very small amount of pitting had occurred in the extreme top of the discharge conduit. Forty-seven days of operation at 9% opening in 1942 had caused the damage shown in Fig. 40.

Parker Dam Spillway Piers

The spillway at Parker Dam has five 50-ft by 50-ft stoney gates to pass the flood waters. These gates were also used for passing the flow of the river during the low-water season, particularly during the early years of operation, before the power plant was completed. As a result of this early scheme of

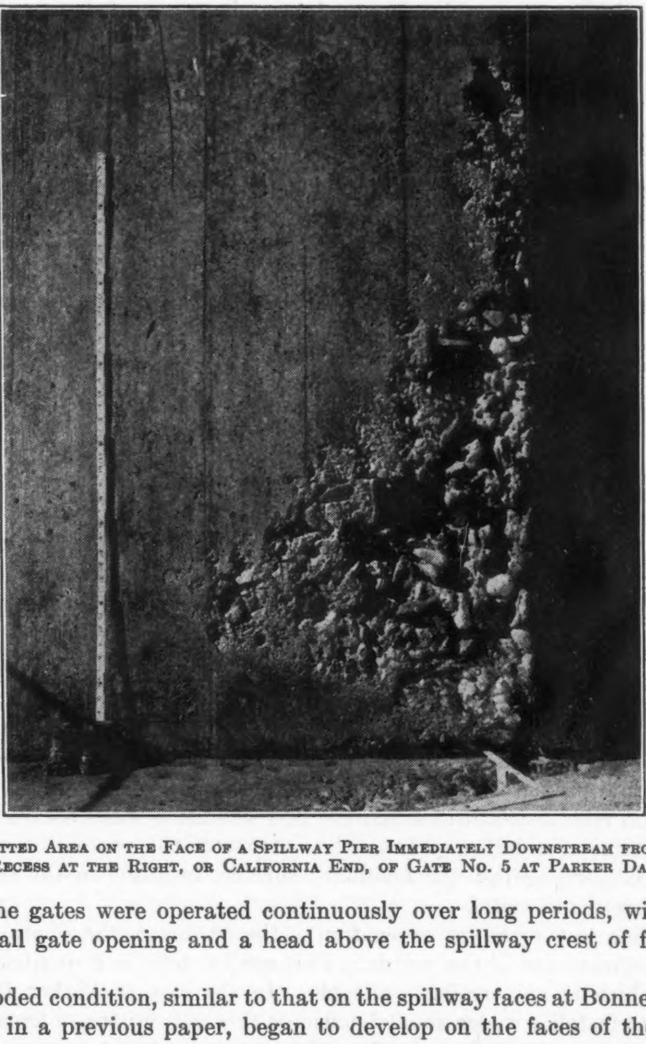


Fig. 41.—Pitted Area on the Face of a Spillway Pier Immediately Downstream from the Gate Recess at the Right, or California End, of Gate No. 5 at Parker Dam

release, the gates were operated continuously over long periods, with a relatively small gate opening and a head above the spillway crest of from 40 ft to 50 ft.

An eroded condition, similar to that on the spillway faces at Bonneville Dam described in a previous paper, began to develop on the faces of the spillway piers and on the spillway crest immediately downstream from the gate slot

(Fig. 41). It first appeared below the gate which had the longest record of operation, but there was evidence of it downstream from the other gates. Subsequent operation of the other gates had gradually developed the same pattern on all ten of the pier faces in lesser degree of intensity. Photographic inspection at approximately yearly intervals discloses some increase in the extent of the area and depth of the erosion, but not sufficient to cause undue alarm, particularly since the power plant has been placed in operation and most of the low-water flow passes the dam through the turbines.

Model studies were undertaken to reveal the cause and means of eliminating pitting at Parker Dam and to prevent it at future installations. Incompleted studies, including all possible circumstances, have revealed several points of interest. The use of transparent models revealed cavitation under the end of the gate in the gate slot as a result of a vortex. Pitting, caused by the collapse of the low-pressure pockets breaking away from the bottom of the vortex, is the only logical explanation of the damage to the pier face.

A similar installation at Guernsey Dam, in Wyoming, showed no signs of erosion on the spillway face even though the gate has operated in the same range for a long period of time. This naturally raised the question as to why erosion occurred at Parker Dam and not at Guernsey Dam.

The gate slot at Guernsey Dam is considerably larger (Fig. 42) in horizontal cross section than the gate slot at Parker Dam. As a result, the vortex in the

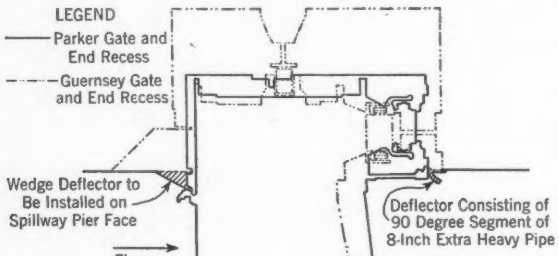


FIG. 42.—COMPARISON OF THE GATE RECESSES OF GUERNSEY DAM AND PARKER DAM, SHOWING THE LOCATION OF THE DEFLECTORS PROPOSED AS REMEDIAL MEASURES AT PARKER DAM

gate slot at Guernsey Dam was large and slow in rotation with no appreciable reduction in pressure at the core, whereas with the smaller cross section of the gate slot at Parker Dam, the angular velocity was high with a very small core and very low pressures in the core. As in the case of the cavitation zone downstream from a venturi throat, the flow condition was unstable and low-pressure pockets broke away from the bottom of the vortex. Some of the pockets collapsed against the boundary surface, resulting in the destruction of the concrete and metal.

According to the present conception of the condition, the solution appears to be the elimination of the vortex. This can be done in a number of ways, none of which is universally applicable. In the case of Parker Dam, it is proposed to install a wedge-shaped deflector (Fig. 42) upstream from the gate sufficient in extent to deflect the flow of water under the gate away from the

downstream corner of the gate slot, thus negating the formation of the vortex. An additional curved deflector consisting of a 90° segment of an 8-in. extra heavy pipe fastened to the metal at the downstream side of the gate slot will further deflect the flow away from the pier face and provide aeration down to the spillway crest. Another solution, practicable where the spillway crest is sufficiently far above the tailwater to provide drainage, is the extension of the end beams of the gates down into wells in the spillway crest, thus making them continuations of the gate slots. These gate-beam extensions will then serve as followers and will fill the gate slot as the gate is raised, providing continuity of the spillway pier face. In the case of a gate 50 ft high, the follower is considered structurally feasible in lengths to 6 ft. The model studies indicated that a follower length of from 2 ft to 3 ft is all that is necessary, since the occasion and duration of operation at the larger openings are infrequent and short.

Insertion of steel plates in the piers in the areas of pitting, as was done on the spillway piers at Bonneville Dam, is also a solution, but one remedying the effect rather than removing the cause.

Boulder Dam Spillway Tunnel

The channel spillway on the Arizona side at Boulder Dam was first placed in operation on August 6, 1941. On August 14, 1941, the drum gates were raised for a few hours and a hurried inspection was made of the tunnel. Little or no sign of erosion was apparent. Operation of the spillway was continued until December 1, 1941, at which time, because of the lowering of the reservoir elevation, it was necessary to start release of water through the tunnel plug outlet needle valves. During the four months of continuous operation, the average flow was approximately 13,500 cu ft per sec, except for several hours on October 28, when one of the drum gates dropped and the maximum flow was 38,000 cu ft per sec.

During a routine inspection of the spillway tunnels on December 12, 1941, an eroded area was discovered in the bottom of the curve connecting the inclined and horizontal portions of the spillway tunnel (Fig. 43(*a*)). The hole was approximately 115 ft long and 30 ft wide, with a maximum depth of 45 ft below invert grade.

The repair (Fig. 43(*b*)) of the damaged area has been described elsewhere (28). The chief concern here is an attempt to analyze the cause of the erosion. A number of theories have been advanced. In the opinion of the writer, the primary cause was misalinement of the tunnel a few feet upstream from the upper end of the eroded area. With an extremely high velocity down the inclined portion of the tunnel (at least 150 ft per sec), the stream followed the invert profile down to the hump; but as it flowed over the hump, the water could not follow the sudden change in grade and a cavitation region formed between the sheet of flowing water and the concrete. The pressure in that region was the vapor pressure of the water, but since this condition was very unstable, the low-pressure pocket or cavity intermittently passed downstream

in the region of higher pressures, collapsed and disintegrated or pitted the concrete as shown in the foreground of Fig. 44. The misalinement is defined by the position of the rope in Fig. 44.

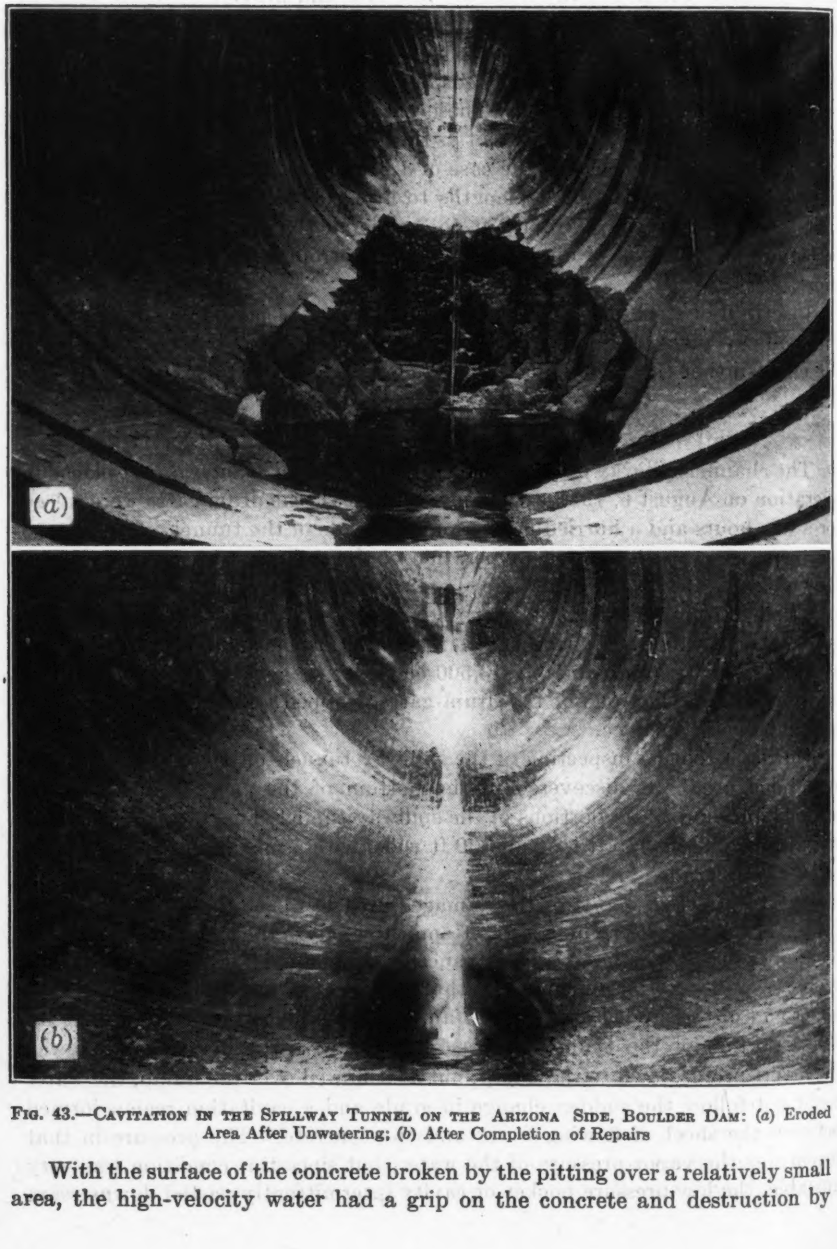


FIG. 43.—CAVITATION IN THE SPILLWAY TUNNEL ON THE ARIZONA SIDE, BOULDER DAM: (*a*) Eroded Area After Unwatering; (*b*) After Completion of Repairs

With the surface of the concrete broken by the pitting over a relatively small area, the high-velocity water had a grip on the concrete and destruction by

impingement started. Imperfections in the concrete, such as rock pockets, cold joints, porous areas, lack of bond, etc., all made the concrete more vulnerable to this attack by impingement. Furthermore, the impingement of the high-velocity water on any exposed joints would cause the energy in the water to be converted from velocity head to pressure head. This pressure was probably transmitted through the planes of weakness in the construction joints

FIG. 44.—PITTED SURFACE DOWNSTREAM FROM THE MISALINEMENT IN THE TUNNEL AT THE UPSTREAM END OF THE ERODED AREA

caused by lack of proper horizontal joint cleanup prior to placement of new concrete. The concrete, being weak in tension, was dislodged in a manner similar to freezing of concrete and the resulting expansion. The concrete was probably dislodged in quite large pieces. After the concrete lining was ripped away, the shattered rock in an underlying fault was dislodged and transported away by the water. The shattered rock in the fault contributed to the extent of the erosion and not to the cause. After the surface was broken by the pitting and the joints were exposed to direct impingement, the sheet of high-velocity water down the tunnel invert acted as a mammoth hydraulic giant.

The pitting of the concrete surface downstream from the hump is analogous to a flesh wound. Infection followed which was aggravated by the weaknesses

in the concrete and the shattered condition of the underlying rock. Under the conditions of misalinement which existed at a critical position in the inclined tunnel, it is doubtful that any material could have withstood the effects of cavitation indefinitely. Of course, perfectly sound homogeneous concrete and underlying rock would have greatly reduced the extent of the erosion.

If the rock pockets, cold joints, and other porous areas in the invert are assumed to be the primary cause of failure, it is difficult to explain why the rock pockets immediately above and below the hump have not been the source of erosion, since the velocity of the water at all three points is for all practical purposes the same. Actually, the coat of black waterproofing and mineral deposit was intact in many places, showing no effect of direct scouring by the high-velocity water immediately above and below the eroded area.

In making the repairs to the tunnel, aside from providing concrete having the most suitable qualities practicable, extra effort was made to provide a smooth continuous surface with no humps or depressions. Two major humps and several minor humps in the invert above the eroded area were entirely eliminated by bushing and grinding, using a template cut to the true radius of curvature. Rock pockets were cleaned, patched, and then ground to conform to the surrounding concrete. Accumulations of grout and mineral deposits were removed. The surface of new concrete in the eroded area was finished carefully to produce a sound, continuous, uninterrupted surface. The surface was given a final grinding with a small terrazzo machine to remove board marks and objectionable offsets, leaving an extremely smooth surface. Minor bulges in the surface were removed by bushing followed by grinding, using a template cut to the correct radius of curvature. Considerable care was used in grinding the surfaces adjacent to the old concrete lining to remove all offsets and other irregularities.

Conclusion

These illustrations are typical situations which should be avoided by designing engineers. Other such examples must exist. If these could be brought to light and explained in the discussions of this Symposium, they would be a definite contribution. Since experience seems to be the principal source of knowledge, those of the profession who are intimate with the effects, even though they have attained that knowledge the hard way and in some cases the embarrassing way, should impart their experiences so that a wide variety of instances can be available to avoid repetition in the future.

EXPERIENCES OF THE TENNESSEE VALLEY AUTHORITY

By George H. Hickox,[5] M. Am. Soc. C. E.

Synopsis

This paper describes cavitation damage experienced by the Tennessee Valley Authority (TVA) in the sluices of Norris Dam, the repairs made to the damaged areas, and the steps taken to prevent similar damage in other structures built by the Authority.

Cavitation at Sluice Entrance, Norris Dam

As of July, 1945, TVA had experienced cavitation in only one of its structures, Norris Dam, in Tennessee. The sluices through the base of Norris Dam are similar in design to those of Madden Dam, in the Isthmus of Panama, and were under construction at the time the damage to the Madden sluices was discovered. In an attempt to prevent similar damage to the Norris sluices, the entrances were bellmouthed by the addition of a half-doughnut-shaped structure on the upstream face of the dam. Fig. 45 shows vertical and hori-

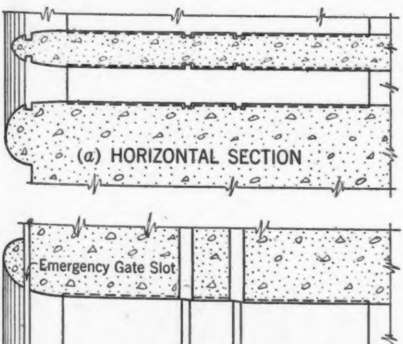


Fig. 45.—Sections Through Sluice Entrances, Norris Dam

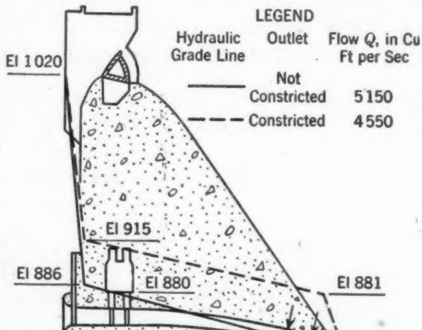


Fig. 46.—Effect of Outlet Constriction on Hydraulic Grade Line

zontal sections through the Norris sluice entrance. The conduit is 5 ft 8 in. wide and 10 ft high and is lined above and below the gates with "semi-steel" (high-test gray iron). Below the liner the conduit was laid out along the trajectory of the jet so that the discharge would enter the bucket on a tangent.

[5] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

Fig. 46 shows the hydraulic grade line for a discharge of 5,150 cu ft per sec and indicates that the pressure in the vicinity of the gates is relatively low, a condition favorable to cavitation. It will be noted that the low outlet contributed to the low hydraulic grade line at the entrance.

When the sluices were placed in operation with the pool at El. 1028, at the time of the release of flood water in 1937, with gates fully open, a pronounced cracking or snapping sound was heard. It seemed to originate somewhat upstream from the operating gallery in the vicinity of the sluice entrances. When the gates were closed 0.4 ft, giving a net opening of 9.6 ft, the noise stopped. It is believed that this noise was caused by cavitation occurring in the sluice entrance. An inspection made by a diver in January, 1938, showed that the sluices had not been damaged. The absence of damage, however, does not eliminate the possibility of cavitation. Accurate information on the discharge of the Norris sluices is not available but the best data at hand indicate that, with the reservoir at El. 1028, the discharges for gate openings of 10.0 ft and 9.6 ft were 5,310 and 4,920 cu ft per sec, respectively. Tests on a model of the Norris sluices, made at the TVA Hydraulic Laboratory in Norris, Tenn., showed that, with a discharge of 5,310 cu ft per sec, the prototype pressure corresponding to the model pressure immediately below the emergency gate slot at the roof of the sluice would have been 74 ft below atmospheric if such a pressure were physically possible. They also indicated that cavitation should stop when gate closure reduced the discharge to 4,850 cu ft per sec. The agreement between this value and the probable discharge of 4,920 cu ft per sec, at which the noise stopped, seems too close to be accidental. It is evident from the tests that the half-doughnut-shaped entrance structure was not satisfactory in eliminating cavitation as long as the emergency gate slot allowed the passage of water past the sharp upper corner of the entrance. Further tests showed that the low-pressure area could be eliminated by partial closure of the emergency gate slot.

TABLE 1.—EVIDENCE OF DAMAGE CAUSED BY CAVITATION, SLUICE LINERS, NORRIS DAM, IN TENNESSEE

Sluice No.	(a) OBSERVATIONS OF APRIL, 1937					(b) OBSERVATIONS OF FEBRUARY, 1938				
	Hours of Operation at the Following Gate Openings, in Feet:				Depth of pitting below gate slot (in.)	Hours of Operation at the Following Gate Openings, in Feet:				Depth of pitting below gate slot (in.)
	0 to 5.0	5.1 to 9.3	9.4 to 10.0	Total		0 to 5.0	5.1 to 9.3	9.4 to 10.0	Total	
1	280	51	219	550	Slight	793	523	297	1,613	1/8 [a]
2	403	43	59	505	0	440	54	75	569	1/4 [b]
3	123	26	509	658	1/8 [a]	674	777	1,380	2,831	1/2 [c]
4	285	4	196	485	0	323	9	221	553	Slight
5	240	11	93	344	0	866	409	1,562	2,837	1/8
6	220	65	983	1,268	3/8 [d]	659	916	1,578	3,153	3/4 [e]
7	424	46	35	505	0	480	60	41	581	Slight
8	15	6	380	401	Slight	210	212	1,006	1,428	Slight

[a] 1/16 in. to 1/8 in. [b] At a point 3 ft from the floor. [c] 1/4 in. to 1/2 in. [d] Near the top of the gate. [e] 1/2 in. to 3/4 in.

PITTING BELOW SLUICE GATES DUE TO CAVITATION

The sluices are lined above and below the gates with castings of so-called gray iron. In April, 1937, the liners below the sluice gates were inspected for

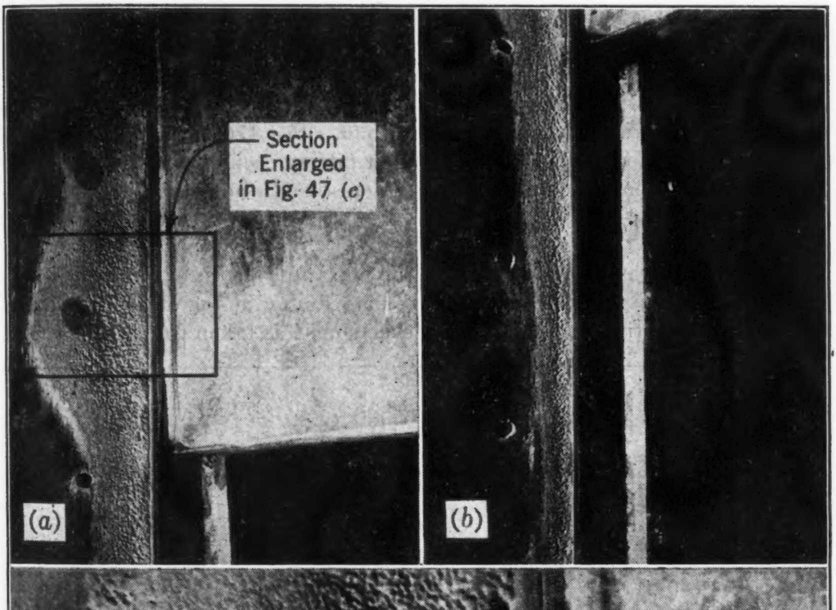


FIG. 47.—PITTING BELOW THE GATE SLOT ALONG THE RIGHT SIDE OF SLUICE NO. 6: (a) UPPER PART OF SLUICE; (b) LOWER PART OF SLUICE; AND (c) ENLARGED SECTION (SEE FIG. 47(a))

evidence of damage due to cavitation. Table 1(a) summarizes the results of this inspection. It gives the hours of operation for gate openings of 0 ft to

5.0 ft, 5.1 ft to 9.3 ft, and 9.4 ft to 10.0 ft, and the depth and location of the deepest pitting below the gate slot. A similar inspection was made in February, 1938, and the results of this inspection are given in Table 1(*b*). The mean velocities during the period of operation varied between approximately 80 ft per sec and 100 ft per sec, depending on the reservoir elevation.

Table 1 demonstrates that the damage to the liner was progressive, increasing with the length of time the sluices were operating. In every case the depth of pitting was greater in 1938 than in 1937. Fig. 47 shows the pitting that occurred on the right side of sluice No. 6. Fig. 47(*c*) is a detailed view of the region indicated in Fig. 47(*a*). Similar pitting occurred on the left side of the sluice. In these illustrations it is interesting to note that, although the liner was pitted to a depth as great as 3/4 in., the bronze gate seat appeared to be undamaged.

An attempt was made to correlate the maximum depth of pitting with the time of operation. Fig. 48(*a*) shows the maximum depths of pitting plotted against the total time of operation for each sluice, as taken from Table 1. In

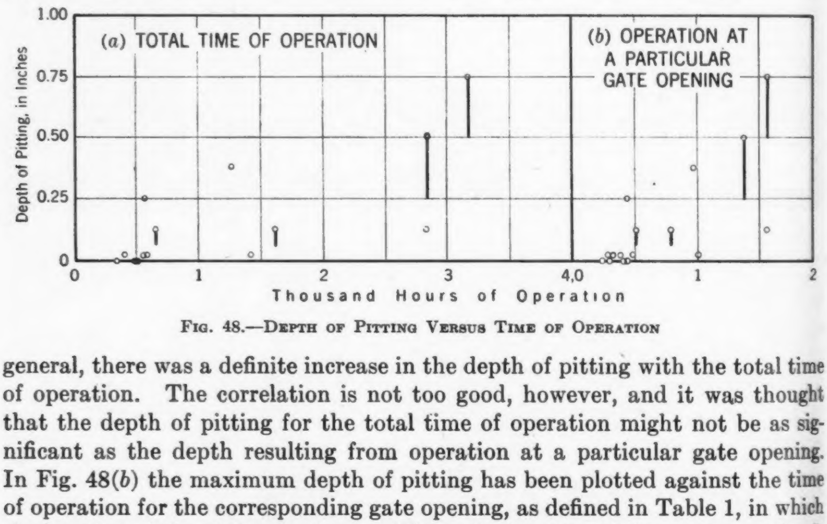
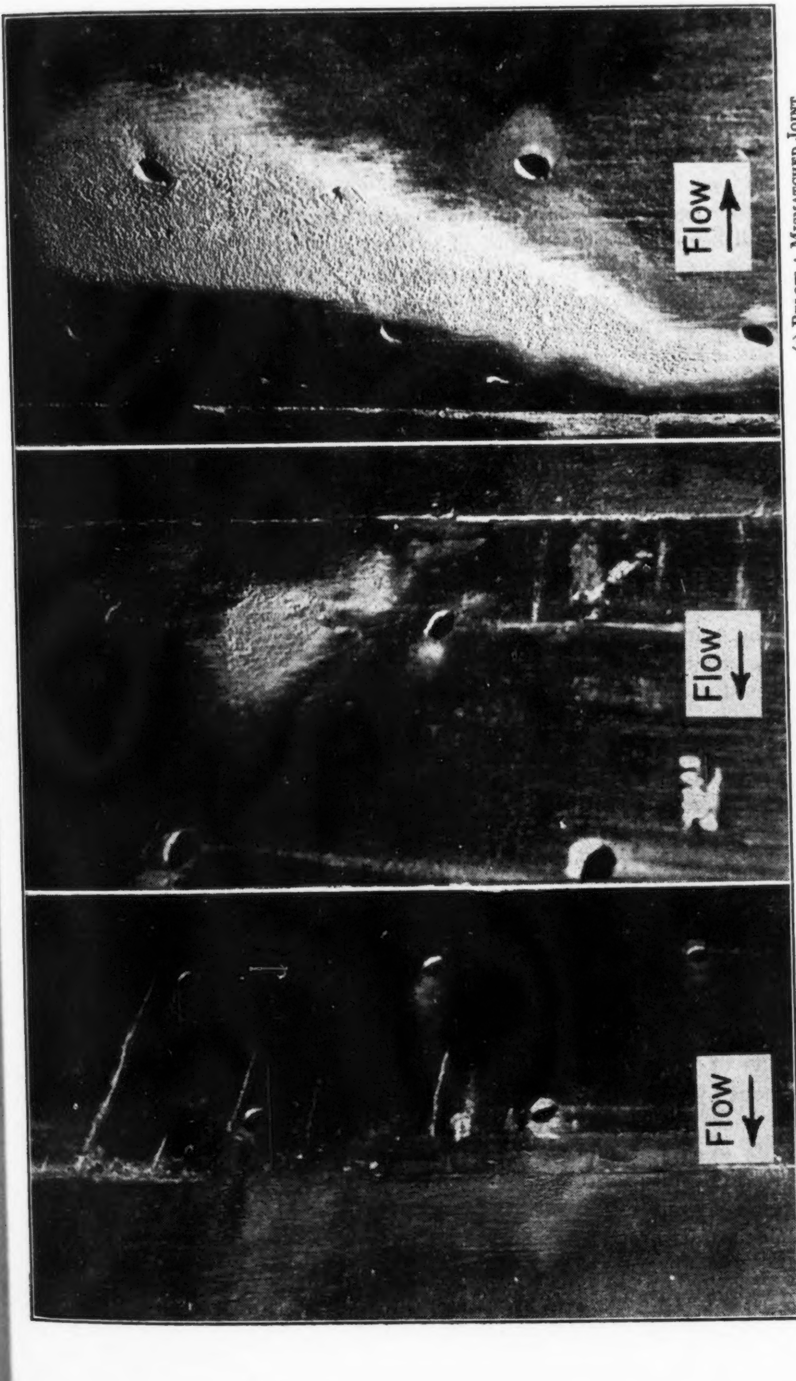


FIG. 48.—DEPTH OF PITTING VERSUS TIME OF OPERATION

general, there was a definite increase in the depth of pitting with the total time of operation. The correlation is not too good, however, and it was thought that the depth of pitting for the total time of operation might not be as significant as the depth resulting from operation at a particular gate opening. In Fig. 48(*b*) the maximum depth of pitting has been plotted against the time of operation for the corresponding gate opening, as defined in Table 1, in which the location of the pitting is given. Where the location of the maximum depth of pitting was not given, it was assumed that the depth was more or less uniform. The depth was plotted against the maximum number of hours for any one group of openings. The correlation does not seem to be much improved. It is probable that the breakdown of operating time is not detailed enough to be significant.

Pitting Below Irregularities in Liner due to Cavitation

Damage due to cavitation was noted not only below the gate slots but also downstream from the weep holes and the irregularities in the joints of the liner sections. Fig. 49(*a*) illustrates the pitting that occurred below the weep holes. Fig. 49(*b*) illustrates the effect of a slight projection into the sluices at one of



(a) Below the Weep Holes (b) Below an Obstruction at a Joint (c) Below a Mismatched Joint

Fig. 49.—Pitting Below Irregularities in the Joints of the "Semi-Steel" (High-Test Gray Iron) Liner Sections

the joints. Fig. 49(*c*) shows the damage that resulted from a slight mismatching of the liner sections at one of the joints. It is evident from these photographs that any irregularities, either depressions or projections, are sufficient to cause cavitation and pitting.

Repairs to Norris Sluices

The inspection of February, 1938, showed that some repairs to the sluice liners were necessary. The repairs were made as follows: All damaged parts that had been pitted to a depth of more than 3/16 in. were chipped out to a minimum depth of 1/4 in. The affected area was then built up to the original surface, by welding, and then ground smooth. The material used was chiefly mild steel but small areas were repaired experimentally with other pitting-resistant materials. Damaged parts that had been pitted to a depth of less than 3/16 in. were repaired by grinding the damaged part and any adjacent bumps or offsets, care being taken to leave only smoothly curved surfaces. Weep holes in the side walls were plugged by mild steel studs, driven tight, cut off flush with the surface, welded around the edges, and ground smooth.

In order to remove one of the causes of cavitation it was decided to raise the hydraulic grade line at the entrance by constricting the outlet. This was done by adding a concrete block 18 in. thick to the top of the sluices at the outlet as shown in Fig. 46. This constriction reduces the capacity of the sluices when operating at full gate opening but has no effect at smaller gate openings. The hydraulic grade line for full gate opening is shown in Fig. 46 for comparison with the original grade line. Three of the eight sluices, Nos. 1, 3, and 6, were constricted in this manner and the remainder were left in their original form.

After a sufficient period of operation, all sluices will be inspected to determine the effect of the constriction in preventing cavitation tendencies and of the repairs in preventing pitting. Sluice discharges since these repairs were made have been very infrequent and no inspection had been made as of July, 1945.

Model Tests for Prevention of Cavitation

Model tests were made of all sluices built by the TVA subsequent to the construction of Norris Dam. These included Hiwassee, Cherokee, Douglas, and Fontana dams.

Hiwassee Sluice.—The sluice through Hiwassee Dam is circular in section with a diameter of 8.5 ft. Ring follower gates were used to eliminate the possibility of cavitation at the gate slot. The hydraulic grade line was raised throughout by constricting the outlet diameter to 7 ft 10 in. The constriction was necessary because the sluices slope downward at a rate of 1 on 5. The possibility of cavitation at the entrance was investigated by means of models. The entrance and a portion of the sluice barrel were built at a scale of 1 : 15. A row of piezometers was placed along the crown of the entrance, special attention being paid to points of change of curvature. The shape of the entrance was a simple bellmouth. Three different forms were tested. The one in which the lowest pressures for the normal range of operation were nearly atmospheric

was selected. Fig. 50(*a*) shows the form of the entrance as it was built. A number of piezometers were installed in the prototype structure in locations similar to those of the model. Observations on these piezometers indicate that the pressures existing on the prototype are very close to those predicted by the model. There has been no indication of any cavitation tendencies. (The dotted lines in Fig. 50(*a*) indicate the presence of a metal liner in this sluice.)

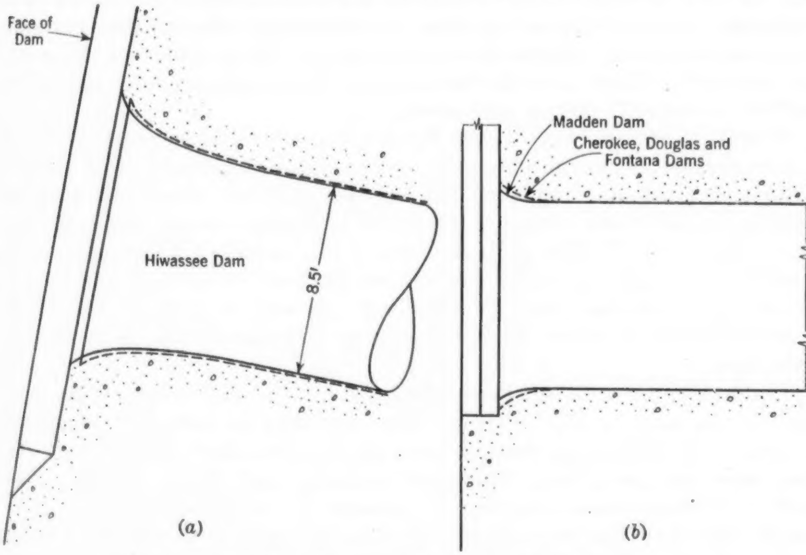


FIG. 50.—COMPARISON OF VERTICAL SECTIONS OF SLUICE ENTRANCES

Cherokee, Douglas, and Fontana Dams.—The sluices through Cherokee Dam, Douglas Dam, and Fontana Dam (see Fig. 50(*b*)) have the same cross section as those at Norris Dam. The Norris gate design was used, making it unnecessary to design new gates for these dams which were built on emergency schedules. The entrances and outlets, however, were modified to avoid the difficulties experienced at Norris.

Tests were made on a 1 : 15 scale model of the entrance at the discharges to be expected in the operation of the dams. Pressures were measured by two rows of piezometers, one row along the center line of the roof and the other in the roof adjacent to the side wall. It was found that the lowest pressures occurred in the corners rather than on the center line. This is interesting in view of the fact that in the damage reported at the Madden sluices, in which the entrance shape was quite similar (see Fig. 50(*b*)), the deepest pitting was on the side walls just below the roof rather than in the roof itself. Since the cross sections of the sluices for Madden Dam and Cherokee Dam are identical, the test results may be accepted as indicative of actual pressure conditions in the Madden sluices.

To produce satisfactory operating conditions on the spillway apron, the outlets for the Cherokee and Douglas sluices curve sharply downward and the

side walls are flared. The area was reduced 15% at the outlet to raise the hydraulic grade line at the entrance. A model of the outlet was built at a scale of 1 : 15 and piezometers were placed in the floor and side walls in the curved part to determine whether pressures might be low enough to induce cavitation. No low pressures were found.

Piezometers were installed in similar locations in the entrance and outlet of both the Cherokee and Douglas sluices to check the results of the model tests. Preliminary observations on the sluices in both dams indicated that a reasonably good agreement with the model tests exists. Final figures on the prototype pressures will not be available until the sluices have operated enough to establish a reliable discharge calibration.

Morning-Glory Spillway—Upper Holston Projects.—The South Holston and Watauga projects have morning-glory spillways discharging into deep vertical shafts and horizontal tunnels. The circular spillway crest was designed according to the results of tests (30) reported by Cecil S. Camp, Assoc. M. Am. Soc. C. E., and J. W. Howe, M. Am. Soc. C. E., in 1939. Piezometers were installed on the spillway face and at various locations in the shaft and tunnel. No negative pressures were found. It is planned to install piezometers at corresponding locations in the prototype structures as a check on the model tests.

Venturi Type Lock Filling Ports—Pickwick Landing and Watts Bar Dams.—Many of the locks on the Tennessee River are filled by means of culverts in the lock walls discharging through ports opening into the lock chamber. In most cases, the ports have bellmouth entrances and flaring outlets. The result is a considerable reduction in pressure at the throat section. It is possible, therefore, that cavitation conditions may exist at the throat of these ports at the beginning of the filling period when the water level in the lock chamber is low. Tests made on the lock at Pickwick Landing Dam showed that the minimum pressure at the throat of one port during filling was 9.6 ft below atmospheric. At the time of the test, the tailwater was 3.0 ft above its expected minimum elevation. The hydraulic grade line at the port throat was 26.6 ft below the water surface in the lock chamber. Model tests on the Watts Bar lock showed that pressures 22 ft below atmospheric existed in one of the ports for a short time at the beginning of the filling period when the tailwater was low. The pressures found in these two locks do not indicate that serious cavitation may be expected, since the minimum tailwater occurs only when the downstream reservoir is being drawn down in advance of floods. Cavitation may thus occur during only a very small part of the time, and it is believed that damage in these locks is negligible or nonexistent. The tests do show, however, that cavitation may occur. This possibility should always be investigated.

Summary

Operation of the Norris sluices indicated that, at full-gate opening, cavitation might exist. Damage to the sluice liners actually occurred wherever the smooth surface of the liner was broken by either a projection or a depression. The depth of pitting was related to the time of operation.

Repairs were made to the metal liner by grinding and welding, using several pitting-resistant materials. Pressures in the affected region were raised by constricting the sluice outlet.

Cavitation was prevented in the sluices of Hiwassee, Cherokee, Douglas, and Fontana dams by making model tests of the sluices and modifying the curvature of the entrance until the pressures at all points were high enough to avoid cavitation. Pressures were also raised by constricting the outlets.

Other structures of the Authority, including the spillways of the proposed Upper Holstón Projects and the ports of the lock filling systems on the main river dams, were investigated for possible cavitation troubles.

BIBLIOGRAPHY

By the Symposium Authors

The following list of references, admittedly incomplete, is offered as a representative bibliography on cavitation in so far as it comes within the scope of the Symposium:

(1) **"A New Method for Studying Cavitation Erosion of Metals,"** by W. C. Schumb, H. Peters, and L. H. Milligan, *Metals and Alloys*, Vol. 8, No. 5, May, 1937, p. 126.

(2) **"Emission of Visible Light from Cavitated Liquids,"** by L. A. Chambers, *Journal of Chemical Physics*, Vol. 5, No. 5, May, 1937, pp. 290–292.

(3) **"Cavitation Research at the Massachusetts Institute of Technology,"** by J. C. Hunsaker, *Mechanical Engineering*, Vol. 57, April, 1935, pp. 211–216.

(4) **"Slow Motion Film of a Cavitating Airfoil,"** by H. Mueller, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 311, 435.

(5) **"Cavitation on Marine Propellers,"** by L. P. Smith, *Transactions*, A.S.M.E., Vol. 59, 1937, p. 409.

(6) **"Cavitation et Tension Superficielle,"** by F. K. Th. Van Iterson, *Proceedings of the Royal Academy of Science*, Amsterdam, Vol. XXXIX, Nos. 2 and 3, 1936; see also *Engineering* (London), Vol. 142, July 24, 1936, pp. 95–96.

(7) **"Theory of Stress Due to Collapse of Vapour Bubbles in a Liquid,"** by R. S. Silver, *Engineering* (London), Vol. 154, December 25, 1942, pp. 501–502.

(8) **"On the Pressure Developed in a Liquid During the Collapse of a Spherical Cavity,"** by Lord Rayleigh, *Philosophical Magazine*, Vol. 34, 1917, p. 94.

(9) **"Investigations into the Causes of Corrosion or Erosion of Propellers,"** by C. A. Parsons and S. S. Cook, *Engineering* (London), Vol. 107, April 18, 1919, p. 515.

(10) **"Tests on Erosion Caused by Jets,"** by E. Honegger, *The Brown Boveri Review*, Vol. XIV, No. 4, April, 1927, pp. 95–104; and Vol. XXI, Nos. 1 and 2, January and February, 1934, pp. 29–30.

(11) **"Rapid Erosion Tests Clarify Wear of Metals,"** by T. F. Hengstenberg, *Steel*, October 17, 1932, pp. 21–23.

(12) **"Cavitation and Erosion,"** by J. Ackeret, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 227, 427.

(13) **"Cavitation Study by the Vibratory Method,"** by H. Peters and B. G. Rightmire, *Proceedings*, Fifth International Cong. for Applied Mechanics, 1938, p. 614.

(14) **"Practical Aspects of Cavitation and Pitting,"** by J. M. Mousson, *Bulletin*, Edison Elec. Inst., September, 1937, p. 373; and October, 1937, p. 423.

(15) **"The Mechanism of Cavitation Erosion,"** by T. C. Poulter, *Transactions*, A.S.M.E., *Journal of Applied Mechanics*, March, 1942, pp. A-31–A-37.

(16) **"Über Werkstoffzerstörung Durch Stosswellen in Flüssigkeiten,"** by J. Ackeret and P. deHaller, *Schweizer Archiv*, Vol. 4, No. 10, 1938.

(17) **"Cavitation Bibliography,"** Rept. R-81 of David Taylor Model Basin, Navy Dept., Washington, D. C. (unpublished).

(18) **"Hydraulische Probleme,"** by H. Föttinger, Berlin, 1926; also **"Experiments on a Few Typical Phenomena of Cavitation,"** by H. Föttinger, *Hydromechanische Probleme des Schiffsantriebs*, Berlin, 1932, pp. 241, 428.

(19) **"Cavitation in Outlet Conduits of High Dams,"** by Harold A. Thomas and Emil P. Schuleen, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 421. (*a*) Fig. 2, p. 425. (*b*) Fig. 13, p. 455. (*c*) Fig. 7, p. 438. (*d*) Fig. 10(*a*), p. 447. (*e*) p. 477.

(20) **"Recent Developments in Hydraulic Laboratory Technique,"** by J. B. Tiffany, Jr., *Proceedings*, 2d Hydraulics Conference, *Bulletin No. 27*, Univ. of Iowa, Iowa City, 1943. (*a*) p. 31.

(21) **"Sub-Atmospheric Hydraulic Models,"** by J. C. Harrold, *Civil Engineering*, June, 1941, p. 361.

(22) **"Cavitation on Baffle Piers Below Dams,"** by H. A. Thomas and W. J. Hopkins, *Proceedings*, 2d Hydraulics Conference, *Bulletin No. 27*, Univ. of Iowa, Iowa City, 1943. (*a*) p. 192.

(23) **"Manual on Lock Valves,"** *Manual of Engineering Practice No. 3*, Am. Soc. C. E., 1930, Appendix 10, p. 77.

(24) *Engineering* (London), November 22, 1940, p. 403.

(25) **"High-Pressure Reservoir Outlets,"** by J. M. Gaylord and J. L. Savage, Dept. of the Interior, Bureau of Reclamation, 1923, pp. 8–9.

(26) **"Experiments Aid in Design at Grand Coulee,"** by Jacob E. Warnock, *Civil Engineering*, November, 1936, p. 737.

(27) **"Distinctive Features of Grand Coulee Outlets,"** by Jacob E. Warnock, *ibid.*, December, 1940, p. 779.

(28) **"Erosion Causes Invert Break in Boulder Dam Spillway Tunnel,"** by Kenneth B. Keener, *Engineering News-Record*, November 18, 1943, p. 762.

(29) **"Bonneville Dam Concrete After Six Years,"** by C. C. Galbraith and R. R. Clark, *ibid.*, March 8, 1945. (*a*) p. 333. (*b*) Fig. 4, p. 334. (*c*) Fig. 5, p. 335. (*d*) Fig. 6, p. 335.

(30) **"Tests of Circular Weirs,"** by Cecil S. Camp and J. W. Howe, *Civil Engineering*, April, 1939, p. 247.

(31) **"Experimental and Theoretical Investigation of Cavitation,"** by J. Ackeret, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, 1930, pp. 1 and 63 (Translation No. 20, U. S. Experimental Model Basin).

(32) **"Cavitation,"** by J. Ackeret, *Handbuch der Experimental Physik*, Band IV, Leipzig, 1931 (Translation No. 22, U. S. Experimental Model Basin).

(33) **"Cavitation Laboratory Practice,"** by L. M. Davis, *Civil Engineering*, March, 1941, p. 148.

(34) **"Developments in Regulating Outlet Valves,"** by G. J. Hornsby, *Transactions*, A.S.M.E., Vol. 64, 1942, p. 85.

(35) **"Determination of the Relative Resistance to Cavitation Erosion by the Vibratory Method,"** by S. L. Kerr, *ibid.*, Vol. 59, 1937, p. 373.

(36) **"Cavitation Research—A Progress Report,"** by C. F. Merriam, presented at a meeting of Hydraulic Power Committee, New England Elec. Utilities Engrs., March 3, 1933 (on file in Engineering Societies Library).

(37) **"Progress in Cavitation Research at Princeton University,"** by L. F. Moody and A. E. Sorenson, *Transactions*, A.S.M.E., Vol. 57, 1935, p. 425.

(38) **"Pitting Resistance of Metals Under Cavitation Conditions,"** by J. M. Mousson, *ibid.*, Vol. 59, 1937, p. 399.

(39) **"Concerning the Influence of Air Content on the Development of Cavitation,"** by F. Numachi and T. Kurokawa, *Werft-Reederei-Hafen*, Vol. 20, No. 4, February 15, 1939, Berlin, pp. 45–49 (translation available at U. S. Waterways Experiment Station).

(40) **"Cavitation and Erosion Investigated as a Problem in Fluid Mechanics,"** by W. W. Pagon, A.S.M.E., 1935 (unpublished; on file in Engineering Societies Library).

(41) **"Design of Hiwassee Dam—Basic Considerations,"** by C. E. Pearce, *Civil Engineering*, June, 1940, p. 340.

(42) **"Testing for Cavitation,"** by R. E. B. Sharp, *ibid.*, May, 1941, p. 308.

(43) **"On the Behavior of Siphons,"** by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

(44) **"Cavitation at Butterfly Valves in Sluiceways,"** by G. Vogelpohl, *Zeitschrift*, Verein Deutscher Ingenieure, Vol. 79, 1935, p. 360.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER

Discussion

BY VITO A. VANONI

VITO A. VANONI,[66] ASSOC. M. AM. SOC. C. E.[66a]—The subject of secondary circulation was discussed at considerable length by several individuals, although it was not considered of primary importance in the paper and little space was devoted to it. The writer especially appreciates the scholarly presentation of the subject by Professor Nemenyi, who covered a tremendous amount of material, some of which is not readily available to American engineers.

Professor Nemenyi advances the opinion that the problem of sediment suspension is essentially three dimensional and that secondary circulation is a factor of prime importance and therefore one which must be studied to insure sound progress in solving the sediment problem. There is no doubt that secondary circulations do exist in sediment-laden flows, and there is also every reason to expect that circulations will affect the sediment suspension process. The important question at this stage of the development of the science is the extent to which circulation affects the phenomenon. If the circulation is of first-order importance, even approximate results probably cannot be obtained without taking the circulation into consideration. On the other hand, there is always the possibility that the effect is relatively minor and that good, workable answers can be obtained from theories that do not give any consideration to the circulation. The engineer, who is interested in obtaining results that can be applied to field problems probably assumes that the circulation is of second-order importance and neglects it until experience shows that circulation must be considered. On the other hand, the problem is intriguing from a purely scientific point of view, and one may be tempted to devote considerable time to it. The writer is inclined to adopt the point of view of the engineer and to

NOTE.—This paper by Vito A. Vanoni was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1944, by Ralph W. Powell, and E. R. Van Driest; February, 1945, by Weston Gavett, and Berard J. Witzig; March, 1945, by A. A. Kalinske; and June, 1945, by Paul F. Nemenyi.

[66] Project Supervisor, U. S. Dept. of Agriculture, SCS, California Inst. of Technology, Pasadena, Calif.

[66a] Received by the Secretary July 2, 1945.

neglect circulation, at least temporarily. Channel resistance to flow, like sediment suspension, also depends on the process of turbulence exchange; yet usable, practical friction formulas, like the Manning formula, have been obtained without considering circulations. Because one flow formula can be used for all cross sections by considering the hydraulic radius as the depth parameter of the flow thus far, at least, it has been possible to neglect the shape factors and resulting differences in circulation and still obtain usable results. This does not mean that in investigations of sediment transportation careful attention should not be given to secondary circulation and to the possibility that it may be a factor of the first-order importance. The attitude is merely that this factor may be neglected until there is proof that it is not negligible. The writer's interpretation is also accompanied by a sincere hope that secondary circulation does not prove to be an important factor. Investigation of this factor would be a very tedious, time-consuming task accompanied by the same difficulties as were the investigations of turbulence which have occupied so many engineers during the past several decades.

Professors Nemenyi and Powell appear to agree that the nonuniform distribution of sediment across the flow is the result, rather than the cause, of circulation, as suggested by the writer. The capacity of a stream to suspend sediment is not uniform across its width. Therefore, the amount suspended at various points in a cross section will be different. Thus, density gradients or forces are produced that can be of appreciable magnitude and will tend to cause correspondingly appreciable cross flows or circulations. These forces will be added to those which normally cause circulation in clear flows and the result will be a modified circulation pattern. The writer's objective in mentioning the subject of secondary circulations in the paper was to call attention to the fact that such density gradients occurred in streams carrying material in suspension and that these gradients could contribute to the circulation. The writer believes that the gradients set up by uneven distribution of sediment may cause circulations even greater than those occurring in clear flows, although this is merely an opinion without any quantitative data to substantiate it.

Professor Van Driest suggests that the discrepancy between the theoretical exponent z, in the sediment distribution equation, and the empirical exponent z_1 may be due, in part, to the transfer of sediment across the flow caused by a concentration gradient in that direction. This is undoubtedly a factor. However, the writer does not believe that the effect would be at all important in measurements made near the center of the flume, since the flume width was almost five times the maximum depth of flow. Professor Kalinske and Mr. Witzig believe that the discrepancy between exponents z and z_1 can be explained by secondary circulations. The writer believes that the discrepancies can be explained in terms of other factors that are more important and that have more effect on the flow. These ideas are outlined in conclusions 1, 2, and 3. Mr. Witzig questions the method of deducing the circulations presented in Fig. 15(c). It will be noted from Fig. 15(b) that the sediment concentrations at about the quarter points are higher than at other parts of the section. Because of the higher sediment concentrations, the fluid at these sections would be heavier than the fluid at other sections and, therefore, it would tend to sink and start the circulations indicated.

From an analysis of the data in the paper, Professor Powell concludes that sediment in the flow tends to reduce the roughness, although no relationship is evident between roughness and sediment load. This conclusion agrees with the ones advanced by the writer. Fig. 24 shows additional data on the effect of sediment on the flow obtained by the writer from experiments made subsequent to those reported in the paper. These measurements were made in the flume with a slope of 0.0025, a bottom roughness of 0.88-mm sand, flow depth y_{max} of 0.295 ft, and varying amounts of suspended load, with a sedimentation diameter, $\bar{D}_s$, of 0.100 mm. Fig. 24(*a*), giving the Manning roughness factor, n, as a function of the average sediment concentration, C_m, shows clearly that the roughness decreases as the load increases. The experiments reported in the paper were made with sediment loads corresponding to the lower concentrations shown in the curve. The roughness factors for these rather small concentrations appear to vary considerably; and, for these curves, it will be noted that the point of the smallest concentration does not fit the general relationship very well. This phenomenon may account for the fact that a relationship between the roughness and the sediment concentration was not apparent from the original experiments.

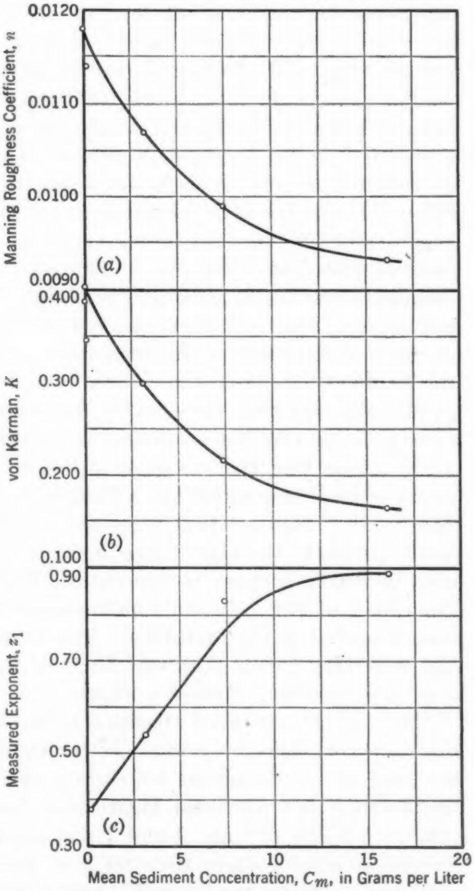


FIG. 24.—EFFECT OF CONCENTRATION OF SUSPENDED LOAD ON FLOW CHARACTERISTICS

The ratios of equivalent roughness to size of bottom sand calculated from Eq. 26*b* for the conditions shown in Fig. 24 are as follows:

Average sediment load C_m in grams per liter	Ratio of equivalent roughness to size of bottom sand
0	0.328
0.17	0.282
3.21	0.190
7.36	0.110
16.2	0.072

The relative roughness decreases very appreciably as the sediment load is increased.

Mr. Witzig suggests that fine sediment deposited between the sand roughnesses cemented to the bottom may account for the decrease in the roughness observed with suspended load. This explanation was considered during the experiments, but it was discarded after further investigation. A careful examination of the flow showed that the sediment load tended to decrease the roughness and to increase the velocity even when very minute quantities of suspended material were deposited on the bed. A more tenable explanation, as outlined in conclusion 6, attributes the decrease in the resistance to the effect of the sediment in damping out the turbulence and thus reducing the eddy viscosity of the flow and the universal constant k in the universal logarithmic velocity distribution formula. This effect is expressed as a reduction in the boundary roughness, although, in a strict sense, the boundary roughness does not change. Instead the change occurs within the body of the fluid. By reducing the intensity of the turbulence, the capability of the flow to transfer the boundary shear to the body of the flow is reduced, with the result that equilibrium between the gravity force tending to cause flow and the shear force resisting the flow is reached at a higher velocity.

Professor Van Driest's observation of the similarity between the velocity profile curves measured by Nikuradse and those reported in the paper is significant. The fact that the semilogarithmic velocity profile graphs do not follow precisely the logarithmic formula means that some judgment must be used in fitting a curve to the data. This may account for the variation in the values of von Kármán's universal constant, k, for clear flows, which has been reported in the literature. The values reported vary from 0.36 to 0.40 and it is easy to see that variations of this magnitude could occur between determinations by different workers.

Mr. Gavett has called attention to the interesting fact that a power law fits the measured velocity profiles better than the logarithmic law. The power law may be advantageous for certain problems but, in general, it is less desirable than the universal logarithmic law. Examination of the measured velocity profiles of Figs. 6 and 7 will show that the logarithmic law fits the measurements for more than 90% of the depth and that the most serious departures occur in the lower 7% or 8% of the depth. The error in calculating the position of the average velocity, due to this departure, is very small, probably much less than errors of measurements; and therefore it is not serious. The difference in velocity between relative depths of 0.368 and 0.393, which are the depths to the average velocity given by the logarithmic law and the one-seventh power law, is only about 1%.

The rather simple looking power law for velocity distribution, when substituted into formulas such as those for sediment suspension, may produce results that are much more complicated than those given by the more general logarithmic law. This is soon realized if an attempt is made to determine the distribution of suspended load, using the power law. The resulting expressions contain the maximum velocity, U_{max}, and the exponent, n, for which analytical expressions do not exist, whereas the logarithmic law gives expressions con-

taining the friction velocity and k, both of which can be evaluated easily. Actually, the relationships between U_{max} and n are contained in the friction formulas which are expressed very conveniently by the more general laws and there is no need to develop the relationships between U_{max} and n.

Mr. Witzig's experience in measuring velocity profiles in the Niagara River cannot be taken as proof that the logarithmic distribution equation does not apply in nature. His experience is contrary to that of John C. Hoyt and Nathan C. Grover, Members, Am. Soc. C. E., who found[67] that the mean depth to the average velocity for 476 measurements in rivers was 0.62 y_{max}. It is possible that Mr. Witzig's measurements were made under conditions of nonuniform flow where the velocity profile could be distorted considerably from the profile expected for uniform flow.

The writer does not agree with Mr. Witzig's statement that the distribution law does not apply to the very fine sediments which are known as the wash load. The distribution of these sediments follows the general trend of the distribution equation, even though quantitatively the equation and the measurements do not agree. This disagreement also exists for the coarser sediments, as has been shown by the writer's experiments and by the experiments of Alvin G. Anderson,[32] Assoc. M. Am. Soc. C. E. In general, for fine sediments the actual distribution is more uniform than that indicated by the equations. In the present experiments, all the suspended load could be considered as wash load because very little of it was to be found in the bed; yet the distribution followed the law very well.

The writer has been asked to comment on the probable effect of uniformly distributed fine sediment in a flow. The principal effect of such a sediment is an increase in the effective specific gravity of the flow. Since the sediment is practically uniformly distributed, its settling velocity or its tendency to settle under gravitational forces is very slight compared to the lifting forces of the turbulence. Because of this fact there will be very little damping effect of the sediment on the turbulence, and in such cases the writer would expect that effects, such as reduction in friction factor and in the von Kármán k, would also be small. Experiments by Professor Kalinske and C. H. Hsia[55] seem to confirm this idea.

It is undesirable to classify the various modes of transportation into bed load, saltation load, and suspended load, because such a classification tends to build up the impression that the modes are fundamentally different. Actually, they are fundamentally the same and are caused by the same kind of fluid forces acting on the sediment. Therefore, the writer prefers to think of the modes as parts of one phenomenon, the only difference being in the intensity of transportation. As has been shown by H. A. Einstein,[68] Assoc. M. Am. Soc. C. E., movement of material on the bed is caused by turbulence fluctuations, which

---

[67] "River Discharge," by John C. Hoyt and Nathan C. Grover, John Wiley & Sons, New York, N. Y., 1912.

[32] "Distribution of Suspended Sediment in a Natural Stream," by Alvin G. Anderson, *Transactions*, Am. Geophysical Union, Pt. II, 1942, pp. 678–683.

[55] "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, *Bulletin No. 29, Studies in Engineering*, Univ. of Iowa, Iowa City, Iowa, 1945.

[68] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–597.

move sediment particles in steps or jumps. As these fluctuations increase in intensity with changing flow conditions, the steps or orbits of the particles increase in length and the sediment is said to "saltate." In the suspension processes, particles that have been lifted off the bed are given further impulses by turbulence in the body of the fluid and they may migrate well into the cross section before they are brought back to the bed. From this point of view, the transportation process is the same throughout its range, the only difference being in the intensity of the transporting forces and their result on the length of step taken by the particles.

Mr. Witzig's statement that the momentum and sediment transfer coefficients are constant over the depth of flow is incorrect. This is evident from Eqs. 21 and Fig. 13, which show that the coefficients have minimum values at the top and bottom and approach maximum values near mid-depth. Fig. 13 also shows that the two coefficients are not equal. In the discussion of Eqs. 2, 3, and 4, the coefficients were expected to differ, although in the development of the theory they were assumed to be the same in order to derive a working theory. In the discussion of the results, the writer has tried to show why the coefficients would not be the same and how this difference would be reflected in the theoretical exponent z and in the measured exponent z_1 for the sediment distribution equation.

Mr. Witzig asks if there was any vertical segregation of the sediment corresponding to the horizontal segregation represented by the sediment clouds or bands. All vertical sediment distribution measurements followed rather closely the form of the distribution formula, Eq. 16, and no discontinuities were evidenced. Since observing these bands of horizontal segregation in the flume, the writer has looked for them in natural streams and canals and has actually seen some bands in flows where the sediment load and depth of flow were small enough to permit seeing the bottom of the flume through the flow. The writer has also seen bands of this kind in dust storms. His observations appear to be contrary to those reported by Mr. Witzig.

The writer agrees with Mr. Witzig's statement that an increase in stream velocity caused by the addition of a sediment load will not result in an increase in sediment-transporting capacity. In discussing this point, the writer called attention to the fact that the depth of flow (and hence the average boundary shear) would decrease as the velocity is increased; and he presented this as another argument to indicate that the sediment-transporting power would not increase with the velocity. The fact that the boundary shear does diminish indicates the probability that the entrainment force will also diminish.

The writer disagrees with Professor Kalinske's statement that no concrete evidence was presented to support the conclusions regarding (1) the effect of suspended load on k and (2) the effect of suspended load in reducing resistance to flow. If Professor Kalinske accepts the measurements as being accurate, then the conclusions drawn by the writer will follow. Admittedly, no evidence is presented to support the conclusion regarding the effect of random fluctuations on suspended sediment. This point was presented to explain the discrepancy between the measurement and the theory and it is believed that the ideas are physically sound and explain the discrepancies qualitatively. The

theory outlined in the paper has certain limitations, but the fact that it gives the correct form of the distribution equation is certainly evidence of correctness. This is also ample proof of the accepted theory that sediment and momentum transfer are similar, although Professor Kalinske apparently does not accept this theory.

The correlation between the horizontal velocity fluctuations U' and the vertical fluctuations V' enters into sediment distribution because in deriving the theory it was assumed that the momentum and sediment transfer coefficients were the same. The expression for the momentum transfer coefficient includes the correlation between U' and V'. Inspection of Eq. 1*a* will show that, to produce a shear, the fluctuations must be of opposite signs; or, in other words, a positive value of U' must be associated with a negative value of V' and vice versa. It is easy to imagine a positive V' occurring when U' is zero. This condition will not contribute to the shear, but it still may cause transfer of sediment. On the other hand, a situation can occur in which momentum is transferred by a V'-fluctuation without transferring sediment because the instantaneous concentration gradient is zero. In this way, one can reason that the coefficients for momentum transfer and sediment transfer need not be the same. Professor Kalinske's example of the condition at the center of the pipe illustrates this fact. Here the shear is zero and hence the correlation between U' and V' is also zero; but the sediment transfer coefficient has a finite value. The idea that the transfer coefficients are not equal was expressed by Theodor von Kármán,[5] M. Am. Soc. C. E., when he remarked that the quantities β, v', and l in Eqs. 2 and 4 which go to make up the turbulent transfer coefficient need not be the same for sediment transfer and momentum transfer.

The writer agrees that, to obtain data on the sediment-transporting capacity of a stream, the bed must be covered with loose material. However, he disagrees with Professor Kalinske's statement that, unless the bed is covered with sediment, the effect of sediment on the hydraulic characteristics cannot be studied. As a matter of fact, having a fixed bed has some definite advantages over having a movable bed, especially in making measurements near the bed. For instance, with a fixed bed the extremely important slope measurement can be made especially well. In this connection, it is possible that the inherent difficulty of measuring the slope with a movable bed may account for fluctuations in k obtained by Professor Kalinske.[55] The total drop in elevation of the bottom of the 80-ft flume, used in Professor Kalinske's experiments, varied from a minimum of $\frac{1}{4}$ in. to a maximum of about 1 in. Accurate measurements of such small slopes with a movable bed would be extremely difficult.

Deposits of some sediment on the fixed bottom of the flume used by the writer may have influenced the bottom roughness, but the amount was never great enough to do more than fill a small part of the space between the fixed sediment, and it is not believed to be an important factor. The strongest evidence of the effect of the sediment on the flow characteristics comes from the measured variations in the von Kármán k in the logarithmic velocity distribution formula. Fig. 24(*b*) offers additional evidence on this point. It

[5] "Some Aspects of the Turbulence Problem," by Theodor von Kármán, *Proceedings*, 4th International Cong. of Applied Mechanics, 1934, pp. 54–91.

will be noted that, as the load increases, k decreases; it is difficult to understand how any sediment deposited on the bottom will affect the value of k. This value can be changed only by modifying the turbulence in the interior of the flow and cannot be changed by merely altering the boundary roughness. The effect is caused by a damping of the turbulence since the turbulence must suspend the sediment against settling. This tends to reduce the momentum transfer coefficients as well as the sediment transfer coefficients. The reduction of the sediment transfer coefficients is reflected in an increase of exponent z_1, as shown in Fig. 24(c).

In comparing the results from Professor Kalinske's experiments[55] with those reported by the writer, the important differences between the two sets of experiments must be borne in mind. The principal differences were:

1. In Professor Kalinske's experiments, the average size of the suspended load was only 0.01 mm, or one tenth the size of the smallest material in the writer's experiments; and
2. In Professor Kalinske's experiments, the bed was always covered with sediment, whereas, in the writer's experiment, the bed was fixed.

With a very fine material, the distribution is practically uniform and the settling velocity of the material is small compared to the lifting ability of the turbulence. Therefore, one would expect that the damping effect on the turbulence would be minor. From this point of view the results of Professor Kalinske's experiments are in agreement with the deductions made by the writer rather than in conflict with them as Professor Kalinske reports. The fact that the von Kármán k, measured by Professor Kalinske, varies as much as it did may be attributed to the difficulties in measuring the slope of the flume with the movable bed.

Professor Powell's questions, regarding the adaptation of Professor Kalinske's turbulence measurements to the problem of sediment transportation, are answered in part in the discussion by Professor Kalinske. Two difficulties occur in connection with using these data: (a) Once these coefficients are measured in a flow, it is still necessary to determine the correlation coefficient β in Eq. 4 before the measured coefficients can be used for calculations; and (b) no readily available analytical expression exists for the transfer coefficient and therefore this coefficient cannot yet be used for the development of a quantitative theory.

Correction for *Transactions:* In June, 1945, *Proceedings*, page 868, Fig. 20(a), change the third line of the legend to read "and (b) Prandtl."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EVAPORATION FROM A FREE WATER SURFACE

Discussion

BY MAURICE L. ALBERTSON, AND G. H. HICKOX

MAURICE L. ALBERTSON,[10] JUN. AM. SOC. C. E.[10a]—Many civil engineers think of the theory of evaporation as applying only to water and of interest to the hydrologist alone. The chemical engineer, however, must understand the vaporization of liquids in drying and evaporating processes and, consequently, has conducted considerable research using other liquids as well as water. Moreover, the phenomenon of turbulence, which plays such an important rôle as the prime mover in the process of evaporation, has been studied in detail by the aeronautical engineer and the meteorologist.

In the section, "How Evaporation Occurs," the author describes briefly the kinetic interpretation of evaporation as used by the chemist. Any molecule that is to leave the surface by overcoming the attraction of the surrounding liquid must possess a minimum upward velocity component, v_o. It follows, therefore, that, as a molecule escapes, the over-all kinetic energy is diminished by $\frac{m\,(v_o)^2}{2}$ so that, when n molecules have departed, the loss of kinetic energy of the system is

$$E_k = \frac{n\,m\,(v_o)^2}{2} \qquad (36)$$

in which m is the mass of one molecule. This loss is equivalent to the heat of vaporization. The rate n_E at which these molecules are leaving the surface may be determined when the system is in equilibrium, because at that time the rate of return of the molecules n_R (max) is equal to the rate of exit n_E—that is, $n_E = n_R$ (max). According to the kinetic theory (63),[10b] the number of mole-

NOTE.—This paper by G. H. Hickox was published in October, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by Carl Rohwer; March, 1945, by C. W. Thornthwaite; April, 1945, by Harry F. Blaney, and Arthur A. Young; and May, 1945, by A. A. Kalinske, and Adolph F. Meyer.

[10] Research Associate, Iowa Inst. of Hydr. Research, Hydr. Laboratory, Iowa City, Iowa.

[10a] Received by the Secretary April 30, 1945.

[10b] Numerals in parentheses, thus: (63), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

cules striking a square centimeter of surface per second is

$$n_R = \frac{p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (37)$$

in which p_v is the partial pressure of the water vapor above the surface in dynes per square centimeter at absolute temperature T in degrees centigrade; n_R is the rate of return of molecules per square centimeter per second; n_R (max) is the limit of n_R when p_v is numerically equal to the vapor pressure p_S of the water; and N_{Bo} is the Boltsmann constant. Therefore, when the system is not in equilibrium—while evaporation is occurring—the net loss of molecules n_L becomes

$$n_L = n_E - n_R = \frac{p_S - p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (38)$$

in which p_S is the vapor pressure of the water at its surface in dynes per square centimeter; and the temperature of the water surface and that of the boundary layer immediately above it are assumed to be the same. It is possible, from Eq. 38, to determine the actual weight rate of evaporation E_w in grams per second per square centimeter by multiplying by m so that

$$E_w = m\,n_L = m\,\frac{p_S - p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (39)$$

Upon applying this relationship specifically to water, cubic centimeters may be conveniently substituted for grams, making E_w numerically equal to centimeters per second. This, in turn, may be converted into inches per day giving

$$E = 2.88 \times 10^5\,\frac{p_S - p_v}{T^{0.5}} \qquad (40a)$$

in which T is now in degrees Fahrenheit, E is evaporation in inches per day, and p_S and p_v are in inches of mercury. Eq. 40*a* demonstrates that the net rate of water transport through the boundary layer varies directly with the difference between the vapor pressure of the water and the partial pressure of the water vapor above it and inversely with the square root of the absolute temperature. As the author has shown, the change in $T^{0.5}$ is relatively small over the temperature range of interest to the hydrologist and, therefore, has little effect on the evaporation rate except as it changes the vapor pressure and partial pressure. One of the limits of Eq. 40*a* (the case of equilibrium) exists when $p_v = p_S$. Furthermore, were it possible to sweep away all water molecules as they leave the surface, allowing none to return, $p_v = 0$ and Eq. 40*a* would reduce to

$$E = 2.88 \times 10^5\,\frac{p_S}{T^{0.5}} \qquad (40b)$$

which is the limit of Eq. 40*a*, giving the greatest rate of evaporation possible at a given temperature. Evaporation as determined by this relation (see Fig. 17) is many times as great as any actually observed. Furthermore, it is much

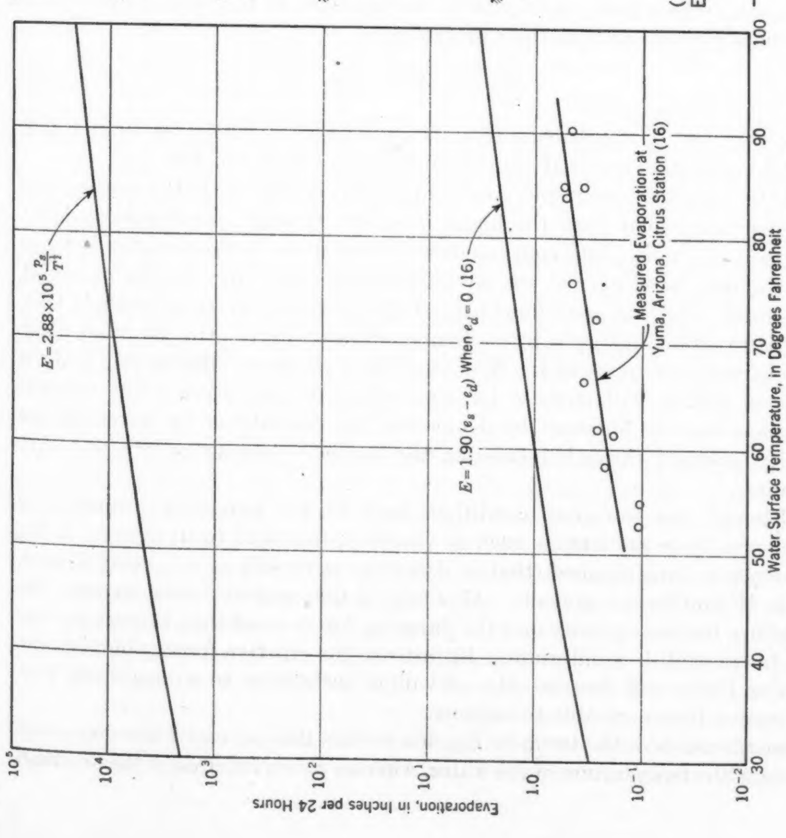


FIG. 17.—RELATION OF MAXIMUM EVAPORATION TO SURFACE TEMPERATURE

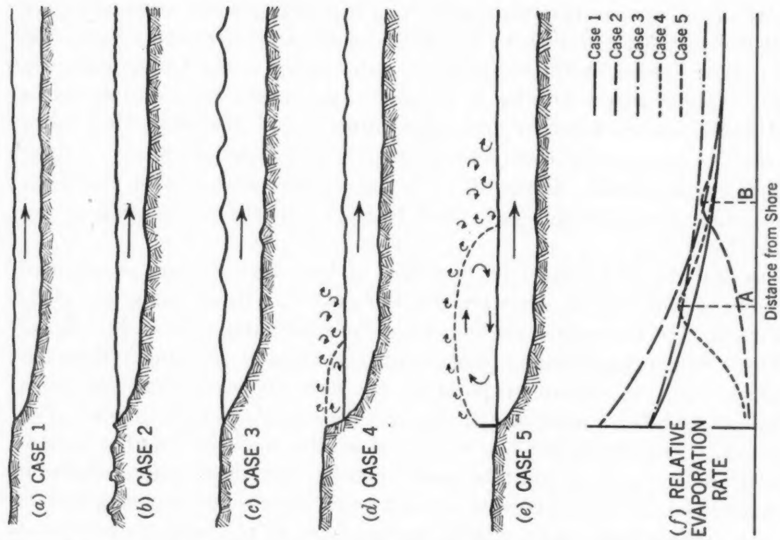


FIG. 18.—RELATIVE EVAPORATION RATES FOR VARIOUS SURFACE CONDITIONS

greater than the evaporation computed with the largest wind coefficient given by Carl Rohwer, M. Am. Soc. C. E. (16*d*). Although this tremendous difference is in part caused by the existence of water vapor in the ambient air, it is primarily explained by assuming, as the author does, that the molecules travel through the boundary layer by molecular diffusion and, therefore, that only a very small percentage of them escape to the turbulent air above without returning to the liquid. Evidently, the natural turbulence over the water surface is never sufficiently great even to approach the elimination of this boundary layer.

There is little doubt that the processes of heat transfer and evaporation have many similarities as described by the author. However, it is well to remember that, as ∂x approaches zero for a constant value of ∂t in Eq. 2*a*, the rate of heat flow will approach infinity; but, according to the kinetic theory as discussed herein, the rate of evaporation can only approach some maximum finite value which is a function of the temperature of the water surface. This discrepancy, on the other hand, is explained by the fact that another form of the heat transfer equation must be used when the heat flow from a surface is being considered. If H is the rate of heat transmission per unit area and T_s and T_v are, respectively, the absolute temperatures of the surface and the air immediately above it, then the equation

$$H = k' (T_s - T_v) \qquad (41)$$

shows that, as T_v approaches zero, H approaches a maximum finite value, thereby completing the heat transfer analogy to follow Eq. 40*a*.

In the foregoing paragraphs consideration has been given to the mechanics of molecular movement from the liquid into, and through, the boundary layer. Of equal importance, although much less understood, is the transfer of vapor from the boundary layer to the air immediately above it. On the one hand, if absolutely stagnant conditions prevail, the boundary layer is infinitely high, and the resultant mixing is by molecular diffusion only. On the other hand, if conditions arise to make the flow unstable, turbulence will develop and the degree of mixing will increase tremendously. At any given point unstable conditions may be obtained by decreasing the viscosity or by increasing the velocity gradient through increasing the ambient velocity or the boundary roughness.

Although the foregoing conditions lead to the perpetual formation of turbulence, there are factors, such as viscosity, that lead to its decay. If the turbulence is being damped (that is, decaying) as rapidly as it is being created, a state of equilibrium prevails. However, if the creative forces increase, the turbulence becomes greater and the damping forces must then become equally large to reestablish equilibrium.. If, instead, the creative forces diminish, the damping forces will decrease the prevailing turbulence to a magnitude that the creative forces are able to support.

Consideration of the terms in Eq. 40*a* reveals that p_S and T are dependent only upon the temperature of the water, whereas p_v is a function of the overhead

turbulence and the vapor concentration and temperature of the ambient air. The turbulence is dependent upon the wind velocity, the viscosity of the air, and the surrounding roughness, such as, for example, the geometry of the evaporation pan, and is therefore dependent upon a characteristic Reynolds number of the flow over the pan. Hence, $p_v = \phi(\mathbf{R}, c_A)$ and Eq. 40*a* becomes

$$E = 2.88 \times 10^5\, T^{-0.5}\, p_S - 2.88 \times 10^5\, T^{-0.5} \phi(\mathbf{R}, c_A) \quad \ldots\ldots\ldots (42)$$

in which $\mathbf{R}$ is the Reynolds number; and c_A describes the temperature and vapor concentration of the ambient air. In Eq. 42, the first parameter 11,330 $T^{-0.5}\, p_S$ is the same as that in Eq. 40*b* and has been determined as in Fig. 17. The second parameter, however, must be determined empirically. The following typical cases serve to illustrate the foregoing principles.

Case 1.—First consider the case of a large lake, bounded on the windward side by a broad smooth plain, with very little difference in the shore and lake elevations. Assume that the surface temperature of the lake is constant, that the general wind velocity distribution over the plain and lake is uniform, and that the ripples on the lake give it the same roughness as that of the land. With these conditions prevailing, the turbulence of the wind near the water surface will likewise be uniform and the only variable factor influencing the rate of evaporation at various points across the lake is the vertical vapor-concentration gradient $\partial c/\partial x$. As shown in Fig. 18, the maximum rate of evaporation will occur at the water's edge; then, as the air immediately above the surface develops a partial pressure of water vapor, the gradient becomes smaller; and, hence, the rate of evaporation asymptotically approaches a minimum value. The total evaporation per unit width across the lake is represented by the area under the curve.

Case 2.—If the land on the windward side is not smooth but extremely rough, the turbulence of the oncoming air will be higher and at the shore the initial rate of evaporation will also be higher than in case 1. However, the turbulence will be partly damped across the lake by the viscosity of the air (the roughness of the lake not being great enough to maintain the initial turbulence) until the turbulence and resulting rate of evaporation approach the same limit as that of case 1—although the total evaporation across the lake is obviously greater.

Case 3.—If the windward land conditions are the same as those in case 1 except that there are waves on the lake, the initial rate of evaporation will be the same as that in case 1 but will approach some higher minimum value as the turbulence reaches equilibrium for the given surface (lake) roughness.

Case 4.—If the conditions are again the same as those in case 1, except that the land is considerably higher than the water surface because of a cliff at the shore, then a large eddy will result at the down-wind side of the cliff; and, along the upper edge of the eddy, considerable turbulence will be created and carried into the main wind stream. Because the turbulence is greater at point A than in case 1, the rate of evaporation tends to be greater; however, this is partly offset by the fact that the vapor concentration has also increased. Hence, the resulting evaporation rate may be greater or smaller than that

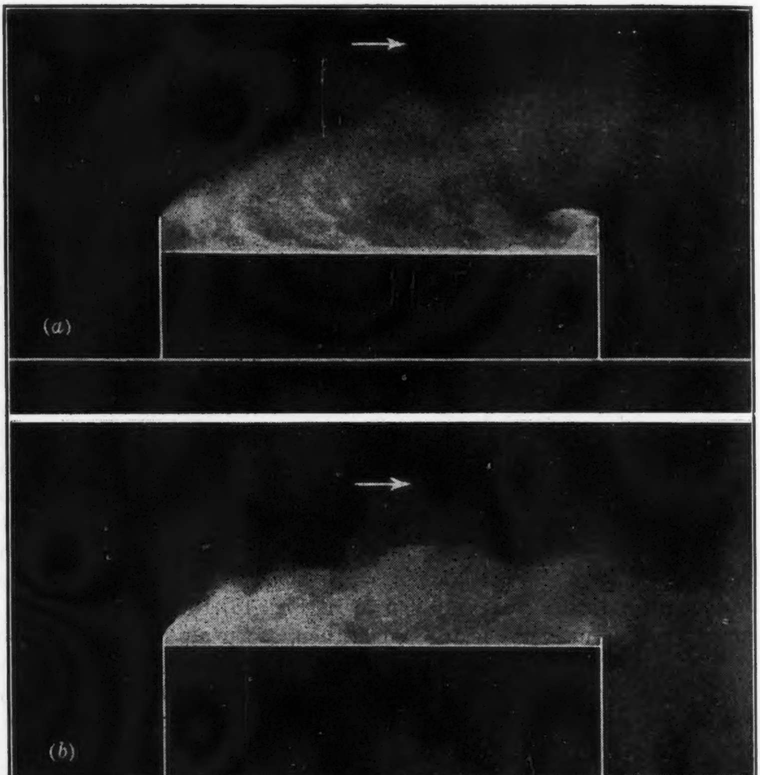


Fig. 19.—Concentration of Smoke over Variously Shaped Schematic Evaporation Pans with the Same Oncoming Wind Velocity

indicated at point A but will approach the minimum value of case 1. Within the primary eddy zone the rate of evaporation is obviously much less than in the previous cases because the eddy tends to recirculate the same air, thereby maintaining a high vapor concentration over this region.

Case 5.—Finally, assume for the sake of argument that the conditions are the same as those in case 1 but that a high wall is at or near the shore; turbulence will be created in much the same manner as in case 4 except that the eddy will extend higher and farther from the shore, causing the greatest rate of evaporation at point B. Again, the effect of increased turbulence is offset partly by a high vapor concentration at point B, and the evaporation curve may lie above or below that shown except that it approaches the same minimum as that in cases 1, 2, and 4.

Of particular concern to the hydrologist is the evaporation from a standard evaporation pan which may be considered as a slightly modified miniature of one or another of the foregoing cases. The turbulence at ground level is determined by the roughness of the terrain and objects immediately upwind from the point in question; and in many cases, such as the Weather Bureau Standard Class A pan, the governing turbulence over the pan is determined by the shape of the pan itself. This is quite graphically seen in Figs. 19(*a*), 19(*b*), 19(*c*), and 19(*d*) which are photographs of the diffusion of smoke over the surface of schematic two-dimensional pans—the concentration of smoke being analogous to the concentration of water vapor over an evaporation pan. To obtain these photographs, titanium tetrachloride was placed on the surface of the pans in a glass-walled air tunnel, 2 in. wide by 22 in. high and 36 in. long. For the range of conditions investigated, the geometry of the pans fixed the scale of the turbulence and the eddy pattern whereas the wind velocity determined only the violence of the turbulence.

As the wall in case 5 creates an eddy, so the side of the evaporation pan creates a similar eddy in miniature. At the left edge of the water surface in Fig. 19(*a*) is a relatively stagnant zone where, as described in cases 4 and 5, there is less evaporation than farther to the right where the concentration gradient $\partial c/\partial x$ is greater. When the freeboard is reduced, as in Fig. 19(*b*), the primary eddy (in these cases extending to the other edge of the pan) is changed only slightly—its size being determined mainly by the height of the pan above ground level. When the pan height is reduced, however (as in Figs. 19(*c*) and 19(*d*) which simulate buried pans), the primary eddy is so small that it covers only a fraction of the pan width, thereby leaving a considerable portion of the surface exposed to the small but violent eddies shown in the photographs. From these photographs it is readily seen that, when the pan is so placed that the turbulence of the air immediately above it is different from that of the surrounding atmosphere, an abnormal evaporation rate will result. Examples of this condition are obtained from placing a standard Class A pan on the surface of level ground or among scattered clumps of underbrush or trees. Evaporation in the first example will be more, and that in the second will be less, than when the turbulence of the air along the ground remains relatively unchanged as it passes over the water surface. Before deciding upon the shape and position of a pan to be used at a given location, consideration should be

given to the general roughness and other features of the surrounding terrain which create turbulence so that a pan which will not cause abnormal turbulence may be chosen.

Many investigators in the past have studied evaporation of different liquids from various surfaces using wind tunnels to control the wind velocity. However, the writer could find no place in the literature where adequate measurement of the turbulence or the velocity gradient above the liquid surface has been described. These variables obviously must be determined for a complete analysis and understanding of evaporation. Thus, there is an excellent opportunity for much desired research in a wind tunnel, designed and operated so that the turbulence of the oncoming air and the eddy form over the liquid may be controlled and measured. There is also a need for wind-tunnel studies of the flow patterns over standard evaporation pans and actual measurement of the evaporation from them.

G. H. Hickox,[11] M. Am. Soc. C. E.[11a]—The discussion evoked by this paper is gratifying. Many valuable additional data have been submitted, particularly in the discussions by Messrs. Young and Albertson.

Mr. Albertson's amplification of the kinetic theory of evaporation is welcome. A detailed mathematical treatment was omitted from the original paper to save space. It is urgently recommended that every engineer who undertakes to study evaporation familiarize himself with the applicable kinetic theory. An excellent treatment by L. B. Loeb (5) is available. If more of the investigators of evaporation phenomena had recognized the validity of the kinetic theory, much more rapid progress would have been made. It should be noted that the equations apply only to momentary or equilibrium conditions and that agreement need not necessarily be expected by trying to use figures based on daily averages.

Mr. Thornthwaite questions the applicability of the analogy between heat transfer and mass transfer. He seems to be of the opinion that turbulent transfer cannot be predicted unless the mass concentration controls the coefficient of turbulent transfer. This objection does not seem to have interfered with the successful prediction of the vertical distribution of silt in a turbulent stream in which the silt concentration has no appreciable effect on the turbulence. The applicability of the analogy can only be judged from the results obtained by its use.

The literature on heat transfer in which over-all transfer coefficients, or film coefficients, have been used is very extensive and forms the basis of present-day studies of heat transfer. Reference is made to such standard texts as those by W. H. McAdams (64) and W. J. King (11). The mass transfer analogy has been used successfully in diffusion problems (13).

Mr. Thornthwaite refers to the observations by Carl Rohwer, M. Am. Soc. C. E. (16), in which vapor pressure of the air was determined within an inch of the water surface by means of an aspiration psychrometer; and he refers to the writer's measurement of relative humidity by means of a sling psychrometer

[11] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[11a] Received by the Secretary June 18, 1945.

as "crude." It has not been generally recognized that the use of the aspiration psychrometer by Mr. Rohwer required a volume of about 9 cu ft of air per minute, which is the rate necessary to maintain the required velocity past the wet and dry bulb thermometers. During an observation period of one minute, therefore, a volume of 9 cu ft of air was drawn through the instrument. This scarcely can be considered as representing a sample one inch from the surface. The writer agrees that the use of a sling psychrometer disturbs still air but he is not sure that the disturbance is much greater than that caused by removing a volume of 9 cu ft from the vicinity of the tests. In the experiments described under the heading, "Evaporation as a Mass Transfer Process: Evaporation into Still Air," the chamber containing the evaporation pan was placed in a room in which the humidity was controlled rather closely and in which the air was continually circulated to maintain a uniform distribution. All measurements within the chamber were made with thermocouples and the air within the chamber was not disturbed except by the evaporation process. Provision was made for circulation of air through the chamber as required by natural convection processes within the chamber. The false floor was placed one-quarter inch from the wall to allow entrance of air and the top of the chamber was open. It was provided with vertical baffles, however, so that the circulation of air in the room outside would not set up disturbances within the chamber itself. The relative humidity was measured outside the chamber and the air within was not disturbed. Under these circumstances the determination of relative humidity can scarcely be considered crude.

Mr. Rohwer questions whether two sets of interference bands might not be obtained from the two surfaces of the mirror A shown in Fig. 3. The production of two sets of interference bands requires much greater optical accuracy than does the reflection of two images. Two sets of bands could be produced only if both the surfaces of mirror A were optically flat and nearly enough parallel to produce interference bands either with respect to each other or mirror B. The interferometer constructed by the author utilized plate glass from a broken automobile windshield. Considerable selection was necessary to find a piece having one surface flat enough to serve. It would be extremely unusual to find the two sides of one piece of glass both flat and parallel. This is a refinement that has not yet been reached in the manufacture of automobile windshields. No difficulty from this source was experienced.

Mr. Rohwer observes that the evaporation below the break in the curve of Fig. 5 is about four times that computed by Eq. 5 and considers that this is an indictment of the equation, because when the water is colder than the air, the cold air above the evaporation pan is stationary and water vapor can escape by diffusion alone. This is not quite correct. Eq. 5 was obtained by analogy with heat transfer from a circular surface into a semi-infinite solid. The analogy is not complete for two reasons. First, the condition of a semi-infinite solid is not fulfilled, as it would require the presence of an infinite mass of cold, perfectly quiet air above the surface of the water. It is clear that the mass of cold air is not infinite. Second, the air is not quiet, as Mr. Rohwer has assumed, but is in motion. Most of the experiments plotted in

Fig. 5 were made on a pan with zero rim height. Smoke injected into the chamber showed that the air mass above the pan settled slowly and spread out on to the false floor around the pan. The result was that the air above the pan was moving slowly. This accounts fully for the increase in observed evaporation over that given by the theoretical equation.

Mr. Meyer's correction regarding the point of measurement of wind velocity used in his formula is gratefully acknowledged.

Mr. Rohwer comments on the application of Eq. 15*b* to diameters greater than 10 ft. Eq. 15*b*, showing the effect of pan diameter on the evaporation rate, was based on heat transfer experiments made on areas having a maximum diameter of probably 3 or 4 ft. Since the value of the exponent of Reynolds' number in Eq. 14 is empirical, one should not expect to find agreement outside the range of experiment. It is perhaps surprising to find that the agreement is as good as it is for diameters up to 10 or 12 ft.

Professor Kalinske concludes that the effect of pan diameter on evaporation varies as the diameter to the -0.125 power. The experimental data available appear to indicate that the exponent is somewhat less than the -0.25 predicted from the mass transfer analogy. Various experimenters have found exponents varying from -0.11 to -0.19.

Mr. Young's observations on the effect of rim height are interesting. They are in direct contradiction to those offered by the writer, but may be explained by the observations reported by Mr. Albertson.

Mr. Young presents additional data on the effect of pan color on evaporation. The effect of pan color appears to be well known but does not appear to be reflected in very many of the published data. The standard galvanized pan changes its surface characteristics very rapidly in summertime because of the growth of algae. It would seem that standard operating instructions for any evaporation pan installation should stress the necessity for frequent cleaning and restoration of the pan surface to a standard condition.

The effect of a screen over a pan in reducing temperature fluctuations is what might be expected. The screen intercepts incoming radiation from the sun during the day and also shields the pan from the low effective space temperatures at night. The result is less absorption of heat in the daytime and a lower maximum temperature, and a reduced back radiation at night with less cooling and a resulting higher minimum temperature.

The photographs of Fig. 19 are valuable additions to current knowledge as to the effect of turbulence. Study of these photographs leads to the conclusion expressed by Mr. Albertson that the surroundings should be taken into consideration in the mounting of a pan. It should be noted also that any pan with a projecting rim possesses inherent boundary effects that should be taken into consideration when an attempt is made to apply the results of pan measurements to large reservoirs where the boundary effects may be insignificant.

Mr. Meyer's statement regarding the absorptive capacity of water is substantiated by Fig. 12, in which the transmission factor is equal to one minus the absorptivity. Fig. 12 indicates that 18% of the incident solar

energy is absorbed in the first millimeter of depth and 31% in the first centimeter. This is a convincing argument for the necessity of measuring water temperatures as near to the surface as possible.

The answer to Mr. Meyer's question about the relative evaporation of a Weather Bureau Standard Class A pan 2 ft deep compared to the 12-in. depth now in use is intimately related to the problem of applying the results of pan observations to reservoirs. The various factors affecting evaporation are so interrelated that it is scarcely possible to change one without affecting all the others. For this reason the writer is not able to give a categorical answer to this question.

The writer regrets that the scope of the paper prevents a discussion of the transfer from pan records to reservoirs. The dissertation filed in the Engineering Societies Library contains additional material which bears on the subject. The problem is complicated and lengthy and has not been worked out in detail. Much additional research will be necessary before it can be solved satisfactorily.

Mr. Blaney asks for detailed specifications of a standard evaporation station. The writer wishes that he was able to present these specifications. Although he has some ideas of what might constitute a suitable installation, it is recognized that they are based largely on incomplete data. The first objective for further study is the establishment of a suitable standard station. Without specifying the details of such an installation it is felt that it should satisfy at least the following six requirements:

1. Water temperatures should be measured as close to the surface as practicable and the measuring device should be sheltered from the direct rays of the sun so that the recorded temperature will be that of the water very close to the surface. The temperature gradient in still water is very high near the surface in daytime and it is the surface temperature that controls evaporation. In large bodies of water that are exposed to wind and wave action the temperature is practically uniform throughout the upper 2 to 5 ft and measurements made a few inches from the surface may be satisfactory. This is not the case with still water pans and this difference should be recognized in the installation.

2. It is probably desirable that the pan be insulated to minimize the boundary effects which do not exist or are negligible in reservoirs. The Weather Bureau Standard Class A pan probably departs more widely from reservoir or lake conditions than any other pan.

3. The pan should be as deep as feasible in order to simulate more closely lake or reservoir conditions.

4. The exposure should undoubtedly be governed by criteria similar to those outlined by Mr. Albertson in his excellent discussion of the effect of exposure on turbulence.

5. Care should be taken to maintain the interior surface so that large variations in rate will not occur.

6. The effect of the fences and instrument shelters in producing turbulence should be investigated and evaluated. It may be found desirable to enlarge

the enclosure and to place instruments with regard to the direction of the prevailing winds in so far as this can be done without shading the pan.

Further research is indicated to establish the type of installation that will be most useful both in furthering the knowledge of evaporation and in supplying useful data to the practicing engineer.

It is hoped that Professor Kalinske and Mr. Albertson will have the opportunity to complete the research outlined in their discussions. If the studies are carefully made and properly interpreted, they will lead to a much better understanding of the problem than has been possible with the type of analyses heretofore made.

No attempt was made to submit a complete bibliography with this paper, and many valuable references were omitted simply because it was not necessary to refer to them. A complete bibliography, existing as it does, in the fields of civil, mechanical, and chemical engineering, meteorology, hydrology, oceanography, biology, and plant ecology, would occupy an entire volume. Two publications, by Mr. Meyer (62) and by A. A. Young, Assoc. M. Am. Soc. C. E. (65), however, contain so many data that they should be included in any comprehensive list.

In conclusion, the writer wishes to express his thanks to those who took time to read the paper and to submit criticism, discussion, and additional data.

Bibliography

(5) **"Kinetic Theory of Gases,"** by L. B. Loeb, 2d Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 106–113.

(11) **"The Basic Laws and Data of Heat Transmission,"** by W. J. King, *Mechanical Engineering*, May, 1932, p. 351.

(13) **"Absorption and Extraction,"** by T. K. Sherwood, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1937.

(16) **"Evaporation from Free Water Surfaces,"** by Carl Rohwer, *Bulletin No. 271*, U.S.D.A., December, 1931. (*a*) p. 9. (*b*) pp. 34–42. (*c*) p. 71. (*d*) Eq. 4*f*, p. 21.

(62) **"Evaporation from Lakes and Reservoirs,"** by Adolph F. Meyer, Minnesota Resources Comm., St. Paul, June, 1942.

(63) **"A Course of Study in Chemical Principles,"** by Arthur A. Noyes and Miles S. Sherrill, 2d Ed., The Macmillan Co., New York, N. Y., 1938, p. 148.

(64) **"Heat Transfer,"** by W. H. McAdams, McGraw-Hill Book Co., Inc., New York, N. Y., 1933.

(65) **"Evaporation Investigations in Southern California,"** by Arthur A. Young, a report of evaporation investigations by the Div. of Irrig., SCS, in cooperation with the Div. of Water Resources, Dept. of Public Works, State of California, Pomona, Calif., April, 1945.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DETERIORATION OF CONCRETE DAMS DUE TO ALKALI-AGGREGATE REACTION

Discussion

BY RALPH R. PROCTOR, MILTON D. BURRIS, H. A. KAMMER, THOMAS E. STANTON, R. W. SPENCER, AND W. L. CHADWICK

RALPH R. PROCTOR,[4] M. AM. SOC. C. E.[4a]—The cracking of concrete from internal expansion has been under investigation by the writer since about 1930, together with the general subject of expansion and shrinkage of concrete in connection with various structures that should be watertight. There seems to have been much more of this type of deterioration in the concrete used in construction since 1930 or 1935 than in older structures. As concrete aggregate probably has not changed much, it would appear that the first place to look for the trouble would be in all the new types of cement that have been produced during recent years; particularly as a considerable part of this trouble has occurred where new types of cement have been used.

Effect of Carbon Dioxide on Various Cements and Its Relationship to Alkali Aggregate Reaction.—During the driving of a long tunnel, large amounts of carbon dioxide were encountered. Tests were made to determine what the effect of this carbon dioxide would be on the concrete tunnel lining and what types of cement should be used to resist this action. As the contract for the cement to line the tunnel had already been entered into, five separate types of cement produced by the manufacturer who was to supply the cement were secured for testing. Small test beams for use in a miniature modulus-of-rupture test were made from sand mixed with each of the five types of cement, two of which were new type cements, and subjected to immersion in water under one atmosphere of carbon-dioxide pressure for eighteen months. At the end of that time it was found that all of the small beams, 1 in. wide, $\frac{1}{2}$ in. deep, and 3 in. long, had expanded and broken into bits, except those made from one type of cement not considered suitable for use in dam construction.

NOTE.—This paper by R. F. Blanks and H. S. Meissner was published in January, 1945, *Proceedings*.

[4] Field Engr., Water System, Dept. of Water and Power, City of Los Angeles, Los Angeles, Calif.

[4a] Received by the Secretary May 7, 1945.

The latter beams were unaffected, having the same strength when tested as similar beams that had been kept immersed in water in the concrete curing room of the laboratory. Thin sections from beams made from each type of cement were prepared and examined under the petrographic microscope. All those that had failed exhibited a white chalky substance between most of the sand grains, but the specimens that had resisted carbon dioxide showed a glassy substance, light amber in color, between the sand grains with only an occasional small speck of chalky substance embedded in the glassy material. Unfortunately, the sample of cement from which these carbon-dioxide resistant beams were made was lost so it could not be tested further.

The conclusion to be drawn from this experiment, in connection with this discussion, appears to be that, if a cement could be manufactured that, when set, would exhibit this glassy carbon-dioxide proof structure, there would need to be no worry about any kind of reaction between aggregate and cement. There would be no travel of chemical compounds, with resulting contact and crystallization, through such a glassy material, as it must have been completely watertight or the carbon dioxide would have broken down the structure as it did in all of the other specimens. Therefore, it appears to the writer that the afore-mentioned carbon-dioxide test of cement should be investigated as a possible means for preventing the kind of concrete deterioration under discussion.

Relationship of Carbon-Dioxide Test to Merriman Cement Test.—Further testing of various cements (about eighty separate samples being tested) for resistance to carbon dioxide disclosed a wide variety of test results which could not be correlated with chemical compounds in the cement or any other cement designations such as sulfate resistant, low alkali, low heat, high early strength, etc.; and only two samples from the group showed reasonable resistance to carbon dioxide. All attempts to correlate test results with any means of testing or designating cement failed, except the comparisons made with the tests proposed by the late Thaddeus Merriman, M. Am. Soc. C. E. (9).[4b] The two cements that resisted carbon dioxide to a reasonable extent were not proposed by any one as carbon-dioxide resistant cements, but had simply been added to the test group because they were cements actually available even though made supposedly for other purposes.

When thin sections of these two best specimens were examined under the petrographic microscope, they exhibited similar glassy characteristics between the sand grains as has been described previously, although there were more small areas of chalky material visible. As far as can be determined, the main difference between the cements that resisted carbon dioxide and those that did not was in the temperature at which they had been burned during manufacture, as indicated by the Merriman sugar solubility index, rather than in chemical compounds. It is of special interest to note that the writer visited a cement mill that was producing a "low-alkali" cement for one large organization which does extensive research in cement; a "sulfate-resistant" cement for another large organization equipped similarly; and a "Merriman" type cement

[4b] Numerals in parentheses, thus: (9), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

for a third large organization; and the exact same clinker was burned and subsequently ground and sold to each of the three separate organizations under their own separate specifications for cement. Furthermore, the cement thus manufactured met exactly all three specifications, according to statements of the manufacturer's chemist.

As far as could be determined, the essential characteristic of this clinker was the temperature in the kilns during manufacture, which appeared exactly in accord with several conversations that the writer had with Mr. Merriman on this subject at the time the tests were being made.

Evidence of Excessive Shrinkage in Expanding Concrete.—Fig. 1 is apparently the upstream or downstream face of a dam. It will be noted that over the distance shown in the picture (possibly 25 ft) there are at least eight vertical cracks that might be $\frac{1}{4}$ in. wide each. If the concrete in the dam had expanded enough to cause such cracking, it would mean that a 25-ft length of the dam had expanded as much as 2 in. (certainly not less than 1 in.) which raises the question of what happened to the dam when it elongated to this extent. Such expansion could occur vertically, of course, but it is difficult to visualize how it could occur horizontally, except in the case of a thin arch dam. Therefore, it would appear that the concrete must have excessive shrinkage properties when dried. This is further evidenced by referring to Fig. 5 to describe a condition noted by the writer on a specimen of concrete from the Parker Dam which had been freshly broken. To illustrate this point, it will be assumed that the piece of aggregate shown in the upper center of Fig. 5 represents the surface of a piece of nonreactive aggregate and that the white spots shown in the picture represent crystal growth on the surface of the nonreactive aggregate and between the aggregate and the remainder of the concrete.

The question arises then as to what circumstances permitted the formation of these crystals between the piece of nonreactive aggregate and the remainder of the concrete, for if the aggregate had been coated originally with cement paste there would be no reason for large quantities of these crystals to form next to the nonreactive aggregate rather than at any other location in the mass of the concrete. Therefore, it appears reasonable to assume that the aggregate probably had a higher coefficient of shrinkage when dried than did the concrete mass, with the result that it pulled away from the surrounding concrete and left a thin void that became filled with solutions that, upon subsequent drying caused the crystallization. Then, upon subsequent wetting, the aggregate expanded, crushing the crystals against the concrete and producing pressure. The formation and shape of the crystals bore out this conclusion. There has been other evidence found by the writer that this type of expansion in concrete can be associated with aggregate having a high shrinkage rate, or, to express it differently, an aggregate which expands considerably if immersed in water when in a dried condition. Therefore, it appears that the shrinkage or expansion properties of aggregates should also be investigated in connection with important structures.

Effect of Adding Admixtures to Concrete.—The free-alkali content of admixtures should also be investigated. It came to the attention of the writer

in connection with one large dam where great care was being used in the selection of low-alkali cement, and where the aggregate was being scrutinized by all known methods, that the admixture used contained enough of the same free alkalies customarily found in cements to more than overcome the benefits secured from using low-alkali cement.

Tests for Reactive Cements.—Test bars similar to those used in autoclave tests were made in an attempt to determine the reactive properties of several types of Portland cement. These were immersed in water and dried through several cycles and the net result secured from the tests was shrinkage rather than expansion, even though the specimens were made with at least 50% of the aggregate and sand comprising supposedly reactive aggregate; however, the cements were not high in the Merriman sugar solubility index.

Conclusion.—It would be most interesting to secure test results similar to those shown in Fig. 7 wherein the expansion of concrete made from the various aggregates would be expressed in terms of the Merriman sugar solubility index of the cements used.

MILTON D. BURRIS,[5] ESQ.[5a]—For several years the writer has used petrographic methods to determine the reactive properties of aggregates with the alkalies contained in Portland cements. Thin sections of proposed aggregates are prepared in the usual manner: A small piece of rock is selected and polished on one side, after which it is cemented with Canada balsam to a glass microscope slide; and the other side of the rock is ground and polished until a thickness of 0.03 mm is secured. The rock section is then nearly transparent and the various minerals contained therein may be identified by polarized or plain light transmitted through the specimen and by the use of other standard petrographic methods for mineral determination.

In the section on "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams," the authors state that

> "Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement to permit positive predetermination of troublesome combinations by petrographic analyses."

The structure of the thin section of rock and the texture of the individual minerals contained therein are usually among the first of the many properties determined by the petrographer. The texture of each specific mineral readily reveals the state of weathering undergone. Since practically all rock specimens show alteration or weathering to some extent, a perfect, unweathered or unaltered thin section of rock is seldom encountered.

This is well illustrated in samples of "andesite" aggregate from the Parker Dam analyzed by the writer. Under the microscope the ground mass (a glassy material filling the space between the larger crystals) in thin sections of these rocks proved to be quite dense and virtually unaltered; however, the andesine crystals (soda-lime-alumina silicates) were almost invariably affected.

[5] Petrographer, Los Angeles, Calif. (formerly with Los Angeles Dept. of Water and Power).

[5a] Received by the Secretary May 7, 1945.

Numerous andesine crystals were poorly formed; others had been altered in varying degrees by chemical action. Many of these crystals showed replacement of portions of the andesine crystals by calcium carbonate to a considerable extent. This is not unusual and is quite common in many rocks where hydrothermal and other forms of metamorphism have taken place. Furthermore, the partial weathering of the aggregate renders it more susceptible to attack by the free alkalies in cement, particularly as some of the silica (SiO_2) had been released in the form of hydrated or opaline silica SiO_2 (n) H_2O—the amount of hydration expressed by n in the chemical formula varying from 1% to 21% and averaging between 9% and 13%.

Reactive Minerals Found in Parker Dam Aggregate.—Although the reactive aggregate in the Parker Dam is generally referred to as an andesite rock, this is partly disproved by the presence in some thin sections of albite and in others of oligoclase as the prevailing feldspar. Albite and oligoclase were also found more or less altered, and when combined with andesine they form the three feldspars that chemically contain the most sodium in the group of six soda-lime feldspars. Thus, it is logical to assume that the further release of sodium from the feldspars as well as from the cement would tend—through chemical reaction with the silica in feldspars and cements—to promote expansion even in a low-alkali cement but at a slower rate than in the case of a high-alkali cement.

Types of Siliceous Rocks that May Be Reactive with Cements.—In the "Summary," the authors state that

> "Aggregates that have been found to react with high-alkali cement include opaline silica, highly siliceous rocks (such as siliceous limestones, chalcedony, and some cherts), and acid to intermediate volcanic rocks."

Wollastonite ($CaSiO_3$) is formed only from a limestone deposit by contact or metamorphism; it is highly siliceous (51.7%) and has a calcium content of 48.3%. This rock has a specific gravity of 2.85 and a hardness of 5, and has been used, when available, with much success as a concrete aggregate. The siliceous limestones to be avoided are those that contain nodules of silica derived from organic matter such as Radiolaria and diatoms. They are readily distinguishable under the petrographic microscope because of the different optical properties of this type of silica. Opaline silica becomes dark under crossed "nicols" (the light passing through the thin section being passed through polarizers placed below and above the thin section and set at right angles to each other), and remains dark as the stage is rotated whereas pure silica, SiO_2, alternates between dark and white for every 90° the stage is rotated.

Additional Precautions To Be Taken in Cement Manufacture.—The author states (see heading, "Corrective and Preventive Measures") that

> "In most cases the reduction in alkali content has been accompanied by marked reductions in C_4AF (tetracalcium alumino-ferrite), which is a fluxing material having little cementing value in itself, and lower C_3A (tricalcium aluminate), which is generally considered to be an undesirable compound."

These two compounds are dependent wholly upon the clay (alumina) and iron content of the materials used in the manufacture of cement. Most of these two compounds are derived from the waste material found in fault zones and from fractures that are always present to some degree in practically all quarries of consequence. These two compounds constitute 20% and more of most cements and range in value from "practically useless" to "quite detrimental." Furthermore, rather than depend upon a coincidence for a reduction of these compounds in the finished product, it is suggested that closer attention be paid to the elimination of waste material from the quarry.

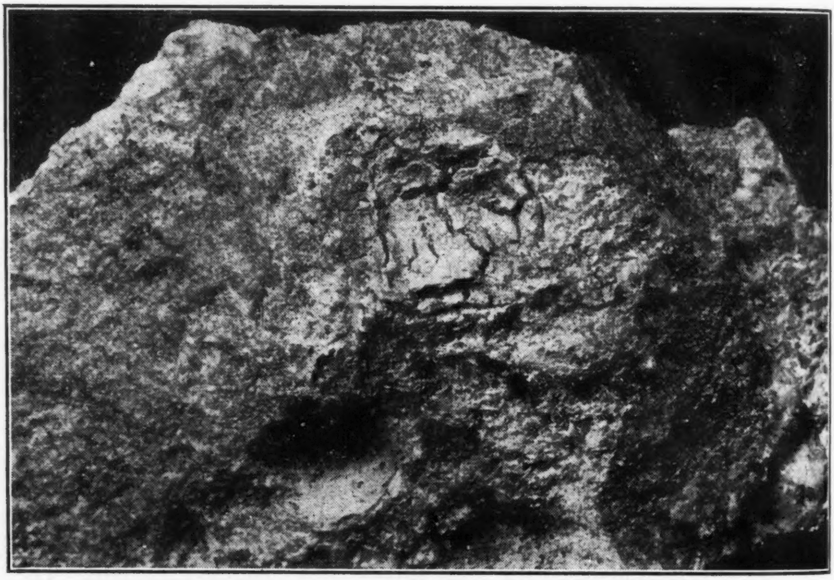


FIG. 11.—UNDERSIDE OF A "POPOUT" AFTER SUBSEQUENT AIR DRYING (Approximate Magnification, 3 ×)

Aggregate Reaction Shown by "Popouts."—Fig. 11 is a view of the underside of a "popout" from the surface of a concrete slab showing a piece of aggregate of unknown composition that became gelatinous through chemical reactions of alkalies and silicates. This chemical change increases the volume of the aggregate particle; and, when the compressive stress thus formed within the aggregate exceeds the tensile strength of the concrete, "popouts" are the result. Popouts occur but an inch or two below the surface of the concrete and over a 3-in. or 4-in. diameter, but the chemical reactions are identical with those which occur at greater depths in mass concrete structures. The shrinkage cracks shown in the aggregate particle near the center of the photograph are the result of dehydration of the gel after exposure to the sun for three or four hours. The dehydrated piece of aggregate shown in Fig. 11 is one of a number of mineral substances that do not yield definite chemical formulas and furthermore show no signs of crystallinity. These substances have been called "gel minerals" or "mineraloids." The power of these minerals to absorb other

substances accounts for their often wide variation in chemical composition although silica predominates greatly in all analyses.

Fig. 1 shows a typical view of concrete where expansion within the mass has occurred. However, if the reaction was entirely between coarse aggregate and cement, there should be evidence of this by popouts caused by pieces of aggregate lying close to the surface. The writer has observed considerable expanded concrete where there were no popouts visible, leading to the conclusion that at least the coarse aggregate was not involved. Accordingly, two thin sections were prepared from a small piece of Parker Dam concrete available from an area adjacent to some expansion cracks having pronounced expansion cracks within one section and slight expansion cracks in the other. There was no positive evidence of reaction between cement and coarse aggregate or between cement and sand, leading to the conclusion that the expansion possibly might be caused by reaction within the cement. It is suggested that the possibility of opaline silica from the limestone remaining in the cement, because of too low kiln temperatures, and uniting with free alkalies remaining in the cement for the same reason, should be investigated thoroughly as a possible source of this expansion.

Conclusions.—All rocks, with the possible exception of pure limestone, are potentially reactive in concrete. This is especially true of any aggregate that shows the least sign of weathering. Incipient alteration in any aggregate chosen should be observed and studied with a polarizing microscope.

Radiolaria and diatoms are microscopic animals and plants, respectively, whose tests or shells are composed of hydrated silica which contains from 1% to 21% of H_2O. Hydrated silicas react with the sodium and potassium hydroxides referred to as free alkalies in Portland cement to form sodium and potassium silicates, thus causing expansion by the formation of a gel.

Far too few data have been made available to engineers concerning reactive aggregates, such as the mineral and chemical composition and the state of weathering, whether incipient or otherwise. For instance, during the process of weathering, an albite feldspar produces a sodium silicate gel and free lime which eventually carbonates causing further expansion. Secondary crystal growth and carbonation of freed calcium, as well as the formation of a highly siliceous ooze or gel, have been observed under the microscope, by the writer, in all expanded concretes examined.

The large amount of concrete manufactured without trouble from expansion appears to indicate that ordinarily both cement and aggregate must contain an excess of available free alkalies to cause expansion and that when either cement or aggregate is practically free of alkalies no serious difficulty has been encountered. However, for an important permanent structure, a full investigation of the free alkali properties of both cement and aggregate should be made, with special emphasis on the analysis of the aggregate by petrographic methods as outlined herein in order that all known precautions may be taken to prevent concrete expansion.

H. A. Kammer,[6] Assoc. M. Am. Soc. C. E.[6a]—The deterioration that develops in concrete structures as a result of the reaction that sometimes occurs between certain concrete aggregates and the alkalies in some Portland cements has been well described in this paper. The authors also have recommended certain corrective and preventive measures that might be used to limit, or possibly to eliminate, the destructive effects of this phenomenon.

In 1941 it was established definitely that the serious deterioration of the concrete structures at the Buck Hydroelectric Plant in Virginia was caused by a reaction that was taking place between the alkalies in the cement and the phyllite aggregate used in the concrete. The basis for this conclusion was reported by R. W. Carlson, Assoc. M. Am. Soc. C. E. (2), and the writer at that time.

In the summer of 1942 work was started on rebuilding and rehabilitating some sections of the substructure of the powerhouse and the exposed surfaces of the dam. In the powerhouse the old concrete structure supporting the turbine and generator was removed to a level a few feet below the bottom of the scroll case and the foundations were rebuilt using cement with an alkali content of approximately 0.5% and an aggregate which the engineers were reasonably certain was not reactive with the cement. The details of this work were reported by Philip Sporn, M. Am. Soc. C. E., and the writer in 1944 (1). Reference points were established in pairs, one of each pair being established in the old concrete which was left in place and the other in the new concrete. The joint between the old and new concrete was treated to break bond.

Rebuilding of the first unit was completed early in 1943 and periodic measurements made since that time definitely indicate that the old concrete adjacent to the machine foundations is still growing. The measurements, made at the four corners of the foundation, indicate some difference in the amount of movement at the several corners which may be due to the variation in the depth of the old concrete still in place below the reference points. Inaccuracies in measurement may also be a contributing factor in these variations. In May, 1945, the surfaces of the old concrete structures were an average of 0.05 in. higher than the new.

The replacement of the foundations for the second and third units has been completed but, although a trend in the same direction as unit No. 1 is noticeable, sufficient time has not elapsed to permit accurate and conclusive information to be obtained relative to the behavior of the structures at these locations. Further growth of the old concrete was expected and allowed for in the reconstruction of the machine foundations. Therefore, no trouble is expected to develop as a result of the continued growth of the old structure.

The downstream face of the north abutment section of the dam was repaired by cutting off the affected face and replacing it with new concrete provided with regularly spaced control joints which were expected to open as the underlying mass continued to grow and expand. After two years this section of the work was behaving as expected and cracks developed in all of the joints. No random map cracking of the type previously experienced had developed by May, 1945.

---

[6] Constr. Engr., Am. Gas & Elec. Co., New York, N.Y.

[6a] Received by the Secretary May 9, 1945.

Messrs. Blanks and Meissner point out that experience thus far demonstrates that reactive aggregates can be used without danger of developing destructive reactions in the concrete mass if the alkali content of the cement is less than 0.6% $Na_2O + K_2O$.

At one of the hydroelectric developments erected on the American Gas and Electric System in 1938 a dolomitic limestone was used for the manufacture of both the coarse and fine concrete aggregates. At that time nothing was known about any destructive reactions developing between some aggregates and the alkalies in some Portland cements. The cement was purchased under rather rigid specifications but no limit was placed on the alkali content.

When the cause of the trouble at the Buck Hydroelectric Plant was discovered, tests were made to ascertain if the dolomitic aggregate used in the new development was reactive. These tests showed that the particular dolomitic limestone that was used was more reactive than the phyllite aggregate used at the Buck development.

Fortunately, samples had been taken of all cements used in the new structure and they had been carefully stored. Tests were made to determine the alkali content ($Na_2O + K_2O$) which was found to be:

Cement No.	Alkali content (%)
1	0.75
2	0.62
3	0.46
4	0.36

Reference points were established at a number of locations on the concrete structures at the new plant and periodic measurements and observations have been made since 1942. Thus far (1945), no unusual expansions have been measured nor has any random-pattern cracking, which is so typical of alkali-aggregate deterioration, been observed.

THOMAS E. STANTON,[7] M. AM. SOC. C. E.[7a]—The material presented by Messrs. Blanks and Meissner is a valuable and interesting contribution to the exceedingly important topic of the "Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction."

Reactive Minerals.—The authors have well stated:

> "Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement * * *."

This is a fertile field for research.

Opal, Chalcedony, Limonite, Glauconite, Dolomite, and Calcite.—In the early reported California experience, it was definitely determined that opal was the reactive ingredient (4). Test data with 1:2 mortar and high-alkali cement (that is, 1.14% Na_2O), covering four years of observation, are given in Table 2. Because of the high lime-magnesia content, rock No. 28039

[7] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

[7a] Received by the Secretary May 31, 1945.

(Col. 2, Table 2) was classified as a siliceous magnesian limestone (4*a*). The expansion results in Col. 4, Table 2, however, indicate that opal is undoubtedly the reactive constituent of this rock. The long-time (forty-eight-month) tests confirm the original conclusions based on tests of specimens less than one year

TABLE 2.—REACTION CHARACTERISTICS OF MINERALS SIMILAR TO THE CONSTITUENTS OF SILICEOUS MAGNESIAN LIMESTONE (ROCK NO. 28039)

Mineral	PERCENTAGE DISTRIBUTION		Percentage expansion after 48 months
	In rock No. 28039	In each test specimen[a]	
(1)	(2)	(3)	(4)
Chalcedony	15.0	10.0	0.019
Opal No. 1	12.0	2.5	1.574
Opaline chert	1.0	10.0	2.837
Limonite	1.0	5.0	0.012
Glauconite	1.0	5.0	0.010
Dolomite	10.0	10.0	0.007
Calcite	60.0	10.0	0.046
Combined	100.0	10.0	0.776

[a] These are the values substituted for equivalent percentages of the nonreactive sand used in each sand-mortar specimen. Separate bars were fabricated for each of the minerals in Col. 1.

old, thereby indicating conclusively that pure chalcedony, limonite, glauconite, dolomite, and calcite have no abnormal reactive properties—at least in the percentages used in this test series.

Other tests have been made with higher percentages of chalcedony without the development of an expansive reaction, whereas tests on opal-bearing chalcedonic cherts and other cherts have developed expansive results proportional to the percentage of opal. An excellent discussion on this phase was contributed by Roger Rhoades in 1942 (10). Mr. Rhoades stresses the fact that:

> "* * * authoritative conjecture surrounds the possibility that some chalcedony verges toward an opaline character in becoming hydrous, and the question naturally arises as to the possibility of such types also assuming other similarities to opaline silica, perhaps becoming potentially reactive in alkaline environments."

He cites the possibility that:

> "* * * opaline material minutely disseminated in chalcedonic aggregates may escape observation and yet impart opaline properties which appear to pertain to the enclosing chalcedony."

The studies of the rocks found in the coastal region of California, where the opal-bearing chalcedonic cherts have caused so much trouble, check Mr. Rhoades' observations.

Andesites, Rhyolites, Tuffs, and Volcanic Glasses.—The Bureau of Reclamation has definitely traced the cause of the trouble at Parker, Gene Wash, and Copper Basin dams to an andesitic rock. As a result, all andesitic rocks have justly been under suspicion as potentially reactive. This rock type has been observed in the Cowlitz (Washington) gravels and is present in the aggregates

from the Friant and other Fresno County (California) areas along the San Joaquin River. The andesites from these areas have been examined petrographically; and, although there are wide variations as to type, texture, and other characteristics, they all have one feature in common—a glassy groundmass. Specimens of concrete from Friant Dam, the Belmont Traffic Circle in Fresno, Calif., the Summers Creek and Courtright Creek bridges in Washington, and the Parker Dam have been examined to observe which aggregates were reacting. Numerous thin sections were made of various particles that appeared to be participating in gel formation. The phenocrysts of plagioclase, hypersthene, hornblende, olivine, augite, rutile, and a few other less common minerals were, in most cases, unaffected by contact with the cement mortar. In various instances, however, the glassy groundmass showed signs of reaction.

It was decided, therefore, to study the glassy groundmass of various basic or intermediate volcanic rocks further. To insure sufficient uniform material for study, and for fabrication into expansion bars, it became necessary to synthesize a quantity of glass having a chemical composition equivalent to that of an average andesite (11)(12). A small furnace was constructed of firebrick. The finely pulverized charge was first moistened with sufficient

TABLE 3.—Percentage Composition of Synthetic Andesitic Glass

Constituents	Unfused (original mixture)	Fused glass
Silica as SiO_2	54.3	61.8
Alumina as Al_2O_3	17.4	18.8
Iron as Fe_2O_3	3.3	2.5
Lime as CaO	12.8	7.6
Magnesia as MgO	3.9	4.0
Sodium as Na_2O	6.3	3.0
Potassium as K_2O	2.0	1.2
Undetermined	0.0	1.1
Total	100.0	100.0

TABLE 4.—Percentage Expansion of 1:2 Mortars of a High-Alkali Cement (1.07%) and Neutral Russian River Sand Combined with Crushed Synthetic Andesitic Glass

Age (month)	Percentage of Andesitic Glass							
	At 70° F				At 110° F			
	5	10	20	30	5	10	20	30
(a) Unweathered Material								
24	0.020	0.019	0.031	0.055	0.061	0.217	0.351	0.491
36	0.028	0.040	0.080	0.148	0.072	0.241	0.368	0.512
(b) Weathered Material								
3	0.012	0.143	0.063	0.008	0.026	0.046	0.123	0.055
12	0.024	0.218	0.461	0.022	0.054	0.271	0.486	0.418
24	0.026	0.218	0.470	0.391	0.049	0.299	0.549	0.627
27	0.025	0.218	0.472	0.549	0.061	0.334	0.616	0.726

water to mold it, and thus to prevent the flame from blowing the powder away. The charge was made of the straight oxides of aluminum, iron, silicon, and magnesium with equivalent amounts of the carbonates, calcium, sodium, and potassium. (Compounds of the rarer elements that are usually found in andesites, such as titanium, zirconium, phosphorus, manganese, etc., were omitted from the charge.) Chemical analyses were made of the charge before fusion, and of the fused glass. These analyses are shown in Table 3. Reaction characteristics of this glass are given in Table 4.

As soon as melted, the glass was ladled from the furnace; and, after cooling at room temperature, it was crushed and sieved into various sizes for fabrication into expansion bars with both low-alkali and high-alkali cements. The percentage of silica dissolved after twenty-one days in 10% NaOH (using the Bureau of Reclamation method) was 0.9%. Solubility in NaOH by the foregoing method does not appear to be significant, however, except possibly in the case of the highly soluble rocks, in which case the high solubility may be indicative of opal.

Chemical Alteration as an Activator.—Observations of many andesite specimens known to be reactive indicated that the specimens were moderately or partly altered and weathered. This condition was existent prior to fabrication into concrete. Work done by R. D. Evans and Howel Williams (13) in Lassen Volcanic National Park (California) shows a few rocks (with opal as a principal constituent) in the vicinity of hot springs to be highly altered. To simulate the Lassen Park condition, some of the synthetic glass was altered in the presence of SO_2 and water, for comparison of expansive reaction with the fresh glass.

A portion of the untreated synthetic andesite glass, crushed to $-20+50$ size was placed in a long glass tube, moistened with water and subjected to daily evolution of SO_2 from a cylinder. A petrographic study of the (SO_2-H_2O)-treated glass indicated a definite change not only in the appearance of the material, but also in its composition and refractive index. Tests at various ages on the unweathered and weathered material (Table 4) indicate a definite increase in reactivity following weathering. This work should be checked against, or compared with, glasses of other composition. A fertile field is indicated for the investigator who has the time.

TABLE 5.—PERCENTAGE EXPANSION, IN THIRTY-SIX MONTHS, OF ROCK SAMPLES FROM STOCK PILES AT A COMMERCIAL PLANT NEAR FRIANT, SAN JOAQUIN COUNTY, CALIF. (1:2 MORTAR)

Type[a] No.	(a) HIGH-ALKALI CEMENT "GS" (1.07% ALKALI)				(b) LOW-ALKALI CEMENT AS "AS" (0.48% ALKALI)			
	PERCENTAGE OF ANDESITE ADDED TO NEUTRAL SAND							
	10%		25%		10%		25%	
	70° F	110° F	70° F	110° F	70° F	110° F	70° F	110° F
1	0.027	0.102	0.025	0.066	−0.006	0.017	−0.003	0.016
2	0.030	0.506	0.365	0.861	0.000	0.153	−0.015	0.345
3	0.018	0.067	0.019	0.086	−0.004	0.011	−0.004	0.041
4	0.013	0.029	0.014	0.042	−0.006	0.001	−0.005	0.005
5	0.023	0.330	0.029	0.519	−0.005	0.005	0.001	0.011

[a] A mineralogical description of each type of rock is listed in the text.

Experience with the San Joaquin River aggregates from the vicinity of the Friant Dam and several miles lower down at Herndon, Calif., indicated that some of the andesitic rocks in these deposits are highly reactive and some only slightly so (Table 5).

The test data from which Fig. 7 was compiled indicate a definitely reactive andesite. California highway experience at the Belmont Traffic Circle, and on the concrete pavement between Fresno and Herndon is a further demonstration of the fact that at least portions of the San Joaquin River aggregates are highly reactive on occasion. However, because experience indicated that aggregates from this source are not consistently reactive and in some cases not reactive at all, a special series of laboratory tests on samples of rock from the San Joaquin River, which, on superficial inspection appeared to be andesitic in character, was begun several years ago with the results shown in Table 5.

The expansion observations in Table 5 indicate that only one (type No. 2) of the five andesites tested is highly reactive at 70° F but that one other (type No. 5), although not excessively reactive at 70° F, is materially activated at 110° F—thereby indicating potential ultimate excessive reactivity at 70° F. Type No. 2 is reactive at 110° even with the low-alkali cement. That the high results with type No. 2 are not accidental is indicated by the trend of all specimens containing this material in combination with a high-alkali cement at either 70° F or 110° F.

The rock samples in Table 5 were selected from stock piles of coarse aggregate at a commercial plant near Friant. There were no marked visible distinctions between the different samples; and it is difficult, therefore, to estimate the probable percentage of each in the combined product of the plant. That these relative percentages are not constant, however, can be inferred by the difference in performance in different projects using the same cement or cements of similar composition. Obviously, the only safe procedure with such variable quality aggregates is the use of a low-alkali cement as was done as far as practicable by the Bureau of Reclamation in the construction of Friant Dam.

In an effort to determine the reactive ingredients in the San Joaquin River aggregates, a petrographic analysis was made of each of the five rock types tested. The results of these analyses are as follows:

1. *Megascopically, a Dense, Dark-Colored Andesite with a Few Relatively Large Scattered Crystals (Phenocrysts) of Feldspar.*—The rounded pebbles of this type show little or no weathering on the outside surfaces. Under a microscope, the dense groundmass is found to be a fine felt of little rods of plagioclase feldspar arranged in more or less flowing lines with their long dimensions parallel with one another (trachytic texture). Included in the felty groundmass are ferro-magnesium minerals (principally augite), magnetite, and amorphous basic glass.

2. *A Dense, Mottled, Grayish-Green Andesite Which Shows Flow Structure.*—Along numerous fine cracks and planes of weakness parallel to the flow lines the rock has been altered to a reddish-brown material. Microscopic examination shows a felty, locally trachytic groundmass composed of plagioclase microlites (embryonic crystals) with interstitial pyroxene, finely divided magnetite, and basic glass. Portions of the groundmass (from 10% to 20%), particularly along seams and fracture planes, have been altered to a brownish, amorphous, hydrated glass which is probably the mineraloid palagonite.

3. A Dark-Colored, Dense, Porphyritic Rock Which Shows Flow Structure and Is Streaked with Fine Light-Colored Veinlets.—This rock consists of large phenocrysts of feldspar, quartz, and hornblende in a once glassy-flowing groundmass which has devitrified, or broken up into aggregates of quartz and feldspar in excessively minute crystals. Numerous quartz veinlets are scattered throughout the rock. This metamorphosed volcanic rock may be classified as an aporhyolite.

4. A Weathered Andesite Whose Light, Grayish-Brown, Rounded Pebbles Have Fresh Dense Unweathered Cores.—The amount of this weathering varies; some of the pebbles are completely altered, whereas others have only a rim of altered material on the outside. The unaltered central core of the rock consists of lathlike phenocrysts of plagioclase feldspar (andesine) in a felty, occasionally trachytic groundmass of plagioclase microlites set in a random mesh, the interstices of which are filled with augite grains, magnetite, and amorphous glass. The altered portion of the rock consists of corroded phenocrysts of plagioclase in a groundmass which has been almost completely altered to palagonite.

5. A Grayish Andesitic Rock with Prominent Flow Lines and Embedded Eruptive Breccia.—Microscopic examination shows that the embedded fragmental breccia and the matrix are very similar. Both are composed of a finely crystalline net of plagioclase feldspar needles with interstitial pyroxene, magnetite, and glass. Well-developed flow banding is common and alteration to palagonite has occurred along seams in some sections.

Although the cause of the high reactivity of type No. 2 can be traced to the glassy groundmass, the analyses indicate that type No. 4 should have been equally if not more reactive—judging by the petrographic analysis, which classifies it as a weathered andesite in which the weathered or altered portions consist of corroded phenocrysts of plagioclase in a groundmass that has been almost completely altered to palagonite. Since palagonite has been determined to be highly reactive, it was felt that type No. 4 should have shown high, instead of apparently negligible, reactive properties.

Because of the possibility that this was another case in which too great a percentage of a highly reactive ingredient had resulted in no measurable reaction, a new series of tests was begun in which only $2\frac{1}{2}\%$ and 5% of type No. 4 were incorporated with a neutral sand instead of 10% and 25% as in the original series. Although preliminary 28-day test results indicate some greater reactions than do the original tests, they are not sufficient to justify any definite conclusions.

Phyllite.—The phyllite referred to by the authors, the reactivity of which is well illustrated in Fig. 7, was discussed by H. A. Kammer and R. W. Carlson, Assoc. Members, Am. Soc. C. E., in 1941 in connection with a report on the Buck Hydroelectric Plant in Virginia (2). The reaction at the Buck Plant was slow, and the expansion did not become sufficiently extensive to be noticeable until 1922. However, the expansion continued until, by 1941, the dimensions were reported to have increased about 0.5% and to be still growing. A possible explanation of this slow development of the reaction; if the phyllite is at fault, is the fact that this rock was used only in the coarse aggregate

portion of the concrete mix. Crushed to sand size, reactivity becomes apparent at a much earlier period, as demonstrated by the Bureau of Reclamation tests which are checked by some work done by the California Division of Highways.

The authors declare (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams") that "Eminent authorities have stated that all rocks, with the possible exception of pure limestone, are potentially reactive in concrete." This conclusion might, with accuracy, have been expanded to state that any such potential reactivity increases with temperature and that most tests indicate a higher expansion at 110° F and 130° F than at 70° F, thereby indicating the effect of temperature as an accelerator and, even on occasion, as an activator. Concrete temperatures exceeding 100° F are not unusual, particularly in pavements and thin walls subject to the direct rays of the sun in the hot summer temperatures that prevail in many areas in California as well as elsewhere throughout the United States. The effect of temperature has been demonstrated by the writer elsewhere (14).

The authors contend (see paragraph preceding the heading, "Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams") that

> "* * * the mortar-bar expansion test has not been developed so that it can be relied upon as an infallible indication of troublesome combinations of cements and aggregates."

This may be true as far as mildly reactive combinations are concerned. However, the mass of evidence accumulated since 1939 has convinced the writer that not only is the mortar-bar test an excellent indication of reactivity but that it is the only dependable test developed to date.

If the authors had qualified this conclusion by a time limit clause such as "within 28, 60, or 90 days when no excessive expansion develops at earlier ages," the conclusion would have greater validity, as undoubtedly in certain slowly reacting combinations, such as the Buck Dam phyllite, a somewhat longer than normal time is required. However, any excessive expansion at early periods can be considered as a definite indication of reactivity, roughly proportional to the magnitude of the expansion.

It is conceded, however, that absence of early expansion cannot be considered as proof of the nonreactivity of the aggregate from a given source when used with a high-alkali cement. The aggregate, if reactive, reacts very slowly; or, if no reaction is observed, the particular sample under test just happens to contain none of the reactive mineral or minerals normally present. The truth of this last statement is well illustrated by experience in California where the mineral causing the reaction (opal) was sometimes present in such small percentages (less than 1.0% of the aggregate) that reactive particles might be entirely missing from a small-sized test sample secured from a commercial supply.

If (as the authors state) the mortar-bar test cannot be considered a satisfactory acceptance test for either cements or aggregates, the only alternative in the absence of any more suitable test is to limit the alkali content of all

cements as a standard requirement if engineers are to insure against destructive combinations.

The writer agrees with most, if not all, of the authors' comments on the deficiencies in the mortar-bar test but would have no hesitation in being guided by the results thereof in the absence of evidence to the contrary. On the other hand, the authors are certainly justified in their conclusion that

> "* * * because of the highly accelerated nature of the test, excessive expansions may ultimately be indicated with a highly reactive aggregate and relatively low-alkali cements, although field records do not indicate that such combinations will necessarily cause trouble in actual service."

An illustration of the validity of this conclusion is the California low-alkali cement AS (Table 5) which apparently never develops excessive reaction at normal temperatures but is peculiarly sensitive and frequently becomes highly reactive even at temperatures as low as 110° F. It would appear logical, however, to follow the lead of the mortar-bar tests in the absence of any contrary and reliable job experience.

The authors cite the case of the Owyhee Dam aggregate which, although highly reactive in the mortar-bar tests, failed to show much reaction in service during the first five years. Neither the alkali content of the cement nor the amount of cement per cubic yard used in the Owyhee Dam construction is given. Is it not possible that an intermediate alkali cement, slow in developing sufficient reaction to be objectionable, was used at Owyhee Dam? The accelerated laboratory tests were made with a high-alkali cement and, therefore, are truly indicative of the ultimate potential reactivity of the aggregate. They constitute a warning that a low-alkali cement should be used under similar conditions.

Furthermore, the nature of exposure is undoubtedly an important factor in the development and progress of the reaction. Many structures in which reactive combinations are used show early distress in portions exposed to adverse weathering conditions, whereas protected portions of the same structure fail to develop appreciable distress, if any. Therefore, although the structural stability may not be seriously lowered, the evidences of the reaction occur where they are most readily observed or where there is the greatest likelihood of subsequent failure through either alterations of freezing and thawing or attack by alkalies from soils or sea water.

Corrective and Preventative Measures.—The authors' comments and suggestions with regard to the necessity for, and desirable procedure to be followed in, making repairs to damaged structures are excellent, as are their comments regarding precautions to be taken in the original construction in so far as the use of low-alkali cement (0.60% or less) is concerned. In addition, the authors state:

> "Tests have indicated that some Portland pozzolanic cements may be safely used with reactive aggregate. It has also been demonstrated that additions of fine pozzolanic materials to reactive aggregate and high-alkali cement combinations or a replacement of some of the high-alkali cement by the pozzolanic material will reduce the expansion."

With regard to the Fresno pumicite, however, there is some evidence indicating possible objectionable rather than beneficial qualities.

The California Division of Highways has tested a number of finely divided minerals for use as correctives but so far has found only two types of materials which appear to be always truly corrective when ground to −200 mesh and used in practicable amounts. These two materials are the opaline cherts and the calcined Monterey shale of the type used in the pozzolanic type of cement (HP) (4*b*)(4*c*). Cement HP had a relatively high-alkali content (0.78%) but has been entirely nonreactive over a period of six years although the standard cement of the same brand and alkali content is decidedly reactive. Checks have been made by adding this same pozzolana (calcined Monterey shale) to the high-alkali cement GS with uniformly successful results. All other local siliceous powders with the exception of the ground opaline rocks, however, have been found relatively ineffective.

TABLE 6.—EFFECT OF TWO TYPES OF POZZOLANIC ADMIXTURES ON SEVERAL REACTIVE CEMENT-AGGREGATE COMBINATIONS

Cement No.	Alkali (%)	Percentage Expansion in Forty Weeks at 70° F		
		No Admix-	Fresno pumicite	Calcined Monterey shale
1	0.47	0.067	0.162	0.014
2	0.47	0.168	0.183	0.015
3	0.51	0.118	0.108	0.016
4	0.62	0.255	0.123	0.011
5	1.17	0.230	0.221	0.086

Table 6 shows the relative effect of two pozzolanic admixtures, ground to pass a 200-mesh screen. The admixtures were Friant pumicite and calcined Monterey shale. Pozzolanic admixtures equal to about 20% by weight of the cement were substituted for an equivalent quantity of the neutral sand portion of the fine aggregate combination. The mortar mix—cement by weight to fine aggregate combination—was 1:2.25; and the fine aggregate was Perkins sand plus 10% of rock No. 28039. Specimens were cured in sealed containers at 70° F. The cements were from lots reported to have been used in the Friant Dam.

The answer to the erratic effect of the Friant pumicite may lie in the alkali content of this material. Chemical analyses show the following percentages of alkali in the Perkins sand, reactive rock No. 28039, Friant pumicite, and calcined Monterey shale:

Material	$Na_2O + K_2O$
Perkins sand	3.53%
Reactive rock No. 28039	0.19%
Friant pumicite	7.04%
Calcined Monterey shale	1.94%

R. W. SPENCER,[8] ASSOC. M. AM. SOC. C. E.[8a]—Damaging expansion of the concrete is known with certainty to occur at only a few points in major structures of the Southern California Edison system although expansion effects

[8] Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

[8a] Received by the Secretary June 18, 1945.

may exist, masked by surface deterioration from weathering. One of the examples of excessive expansion occurs in the drainage gallery in the base of a dam where a pit-run tunnel muck was used in placing a small section of concrete. The expansion is made evident by the cracking along exposed corners of the concrete. The concrete was placed in 1926 and the cracking first appeared in 1940. As of June, 1945, however, tests had not shown the presence of silica gel deposits typical of alkali-aggregate reaction. The remainder of the dam was constructed of quarried crushed granodiorite and shows no signs of excessive expansion.

A second instance of damaging expansion has appeared in the mass concrete footings of one of several flumes serving one of the Company's Kern River hydro plants. This concrete was placed in 1920 and surface cracking was noted in 1942, although it is probable that it began several years earlier. The footing concrete was made with crushed phyllite obtained from a near-by tunnel dump. The concrete flume box was made from a portion of the tunnel dump which is low in phyllite and high in granitic material, and no excessive expansion has occurred. Tests by the Bureau of Reclamation on a sample of the cracked footings showed the presence of silica gel typical of alkali-aggregate reaction.

A third case of excessive concrete expansion is known to occur in the walls of a gallery through a heavy tunnel plug at the Company's Big Creek No. 3 plant. The concrete was placed in 1923 and surface cracking appeared in 1940. The aggregate was chiefly granodiorite obtained from a near-by tunnel dump. The concrete where the expansion occurred was badly segregated and the expansion is confined to portions of the structure that are high in mortar and low in coarse aggregate.

In none of the three aforementioned cases has the quantity of concrete involved been large enough nor the effects of the expansion been important enough to require repair, but the fact that the expansion did occur made it desirable to test as many aggregates as possible in the region for reactive tendencies. The results of tests on thirty-one aggregates taken from various sources on the Edison system, including the three discussed herein, are given in Table 7.

In no case was the expansion measured in the mortar-bar test high in comparison with the expansion caused by a known reactive aggregate such as the siliceous magnesian limestone supplied by T. E. Stanton, M. Am. Soc. C. E., of the California State Highway Department. However, the Kern River phyllite (KP) used in the footings previously mentioned and the crushed andesite from Italian Creek (IL) showed expansion much greater than normal for nonreactive aggregate. The Italian Creek andesite has never been used in concrete but, because its geological age and vicinity are similar, its action in concrete probably resembles the reactive Friant andesite for which the authors show an expansion curve in Fig. 7.

The expansion of the Kern River phyllite test bars checked the field expansion observed in concrete made with this same material.

The test bars made with the crushed granodiorite (3N) used in the tunnel No. 3 plug, although showing greater than average expansion, did not show enough to put the aggregate definitely in the dangerous class if used with a

high-alkali cement. Cored samples of the tunnel plug concrete when examined by the Bureau of Reclamation and the Portland Cement Association did not show deposits of silica gel characteristic of the alkali-aggregate reaction.

TABLE 7.—TESTS FOR REACTIVE AGGREGATES, SOUTHERN CALIFORNIA EDISON SYSTEM

(Expansion[a] in Millionths of an Inch Per Inch After Approximately 880 Days)

Designation	Aggregate	High alkali[b]	Low alkali[c]	Difference
	U. S. Bureau of Reclamation:			
I	Crushed quartz	250	90	160
	Bear Creek Tunnel; Dump:			
B	Fresh crushed granite	210	110	100
	Florence Lake; Stock Pile:			
FN	Fresh crushed granite	250	150	100
FO	Old crushed granite	170	150	20
	Florence Lake; Jackass Meadow:			
FJ	Natural sand	330	180	150
	Camp 60 Tunnel; Dump:			
	Fresh crushed granite—			
WN	Normal	230	50	180
WA	Aplitic	280	160	120
WG	Gneissic	220	40	180
WD	Partly decomposed	120	0	145
	Huntington Lake Dam; Spillway Stock Pile:			
HO	Old crushed granite	170	60	110
HN	Fresh crushed granite	250	130	120
	Huntington Lake Dam; Old Quarry:			
	Fresh crushed granite—			
HQN	Normal	30	− 40	70
HQA	Altered	140	20	120
	Huntington Lake Dam; Line Creek:			
HS	Natural sand	313	93	220
	Tunnel 2, Adit 4; Dump:			
2N	Fresh crushed granite; normal	90	10	80
2A	Selected granite; altered	170	30	140
2O	Old crushed sand	170	130	140
	Italian Creek:			
IL	Crushed lava; probably andesite	630	20	610
	Italian Bar:			
IS	Natural sand	287	13	274
	Tunnel 3, Adit 3; Dump:			
3O	Old crushed granite	230	10	220
3F	High ferro-magnesian material; granitic	130	− 30	160
3N	Fresh crushed granite	430	170	260
	Tunnel 5, Camp 19; Dump:			
5O	Old crushed sand	200	− 90	290
5N	Fresh crushed granite	160	20	140
	Shaver Lake; Reservoir:			
SS	Natural sand	180	133	47
	Shaver Lake; Quarry:			
SQ	Fresh crushed granite	200	20	180
	Last Hope Bar:			
LS	Natural sand	220	120	100
	Kerckhoff Reservoir; Project No. 4:			
4S	Natural sand	187	60	127
4B	Freshly crushed, slightly altered, granite	270	− 15	285
	Sanger:			
S	Natural sand	320	87	233
	Wishon Marble Claim:			
M	Crushed marble	120	90	30
	Kern River, Adit No. 3; Tunnel Dump:			
KP	Freshly crushed phyllite	790	190	600
	Standards for Comparison:			
R	Known reactive aggregate[d]	8,450	170	8,280
RX	100% siliceous magnesian limestone	327	27	300

[a] Three 1 in. by 1 in. by 6 in. aggregate-cement mortar bars (mix, 2 : 1 by weight) for each condition tested; sealed and stored over water at 100° F, water-cement ratio, 0.57 by weight. [b] Specially manufactured, containing 0.89% sodium oxide and 0.43% potassium oxide. [c] Commercial cement containing 0.18% total alkali as Na_2O. [d] 20% siliceous magnesian limestone and 80% Bureau of Reclamation quartz.

Test bars made from a tunnel dump aggregate similar to that used in the expanding block of the dam noted have shown an expansion greater than the average, but not enough to indicate that the aggregate would be dangerous if

used with a high-alkali cement. It is possible that the test bars made with the granodiorite aggregates used for this work and that used for the tunnel plug fail to show excessive expansion because the action differs from the typical alkali-aggregate reaction. On the other hand, it may well be, as the authors suggest in the section on "Symptoms and Diagnosis," that the present accepted test procedure for alkali-aggregate reaction is not sensitive enough to predict expansions that may take place in structures twenty years or more after they are completed.

W. L. CHADWICK,[9] M. AM. SOC. C. E.[9a]—Engineers concerned with the use of concrete are indebted to Messrs. Blanks and Meissner and to others in the Bureau of Reclamation for their straightforward presentation of troubles with concrete as a result of alkali-aggregate reaction. Because the activities of the Bureau cover many projects scattered over the entire western part of the United States, the experience gained is particularly valuable in determining the relatively few undesirable or dangerous cement-aggregate combinations.

The Southern California Edison Company has a number of dams and major concrete structures in the granitic region of the middle Sierra Nevada where natural sand and gravel for concrete aggregates are scarce. Consequently, it has been necessary for most of the concrete in these structures to be made from crushed granodiorite taken either from quarries or from tunnel excavation dumps. Some concrete in these Sierra structures has stood up well but some shows troublesome surface deterioration. Over a period of twenty years continuing efforts have been made to determine the reason for the differences. These efforts have included many tests and petrographic examinations of aggregates and concretes without positive results. When the alkali-aggregate reaction first became known, these structures were re-examined for evidences of such action and a few probable examples were found. The possibility that some particular mineral in the granodiorite was responsible for such trouble and also for other deterioration in the presence of high alkali was considered, as a result of which a fellowship was established at the California Institute of Technology, at Pasadena, in 1942 to test the susceptibility of these individual minerals to reaction with cement-borne alkalies.

Under the direction of John P. Buwalda and Ian Campbell of the Institute, Lloyd Pray undertook the research including the difficult task of separating enough pure minerals from a 500-lb sample of granodiorite to permit the manufacture of a number of 1 in. by 1 in. by 6 in. mortar bars with various concentrations of each mineral. The average composition of the fresh granodiorite was:

Mineral	Percentage
Plagioclase (andesine)	38
Quartz	24
Potash feldspar (chiefly microcline)	18
Biotite	10
Hornblende	8
Magnetite	1
Sphene	0.5
Zircon	Trace
Apatite	Trace

[9] Mgr., Eng. Dept., Southern California Edison Co., Ltd., Los Angeles, Calif.

[9a] Received by the Secretary June 18, 1945.

Mr. Pray was able to obtain all the minerals listed with the exception of the plagioclase feldspar, zircon, and apatite in purities of 96% or better. The plagioclase was obtained as a concentrate containing 43% plagioclase and 53% quartz with the remainder largely potash feldspar. The minerals tested were used as aggregates in different percentages ranging from 2% to 100%. The remainder of the aggregate consisted of a crushed crystalline quartz furnished by the Bureau of Reclamation. The selected percentages were based on consideration of the natural proportion of the minerals in the rock and also of the amount of mineral concentrate readily available. The test bars were made up in triplicate, using a 1 : 2 mix by weight and high-alkali and low-alkali cements of known analyses. The low-alkali cement contained 0.18% of total alkali measured as Na_2O. The high-alkali cement contained 0.87% Na_2O and 0.47% K_2O. After curing for seven days at 70° F and 100% humidity, the bars were placed in a cabinet, over water, with a controlled temperature of 100° F, and length measurements were made periodically over a period of two years.

The volume changes for the principal series of test bars at an average age of 750 days are compared in Table 8, including the results from the use of the

TABLE 8.—EFFECT OF HIGH-ALKALI CEMENT (H) AND LOW-ALKALI CEMENT (L) ON THE VARIOUS MINERALS IN FLORENCE LAKE GRANODIORITE

(Expansion[a] in Millionths of an Inch Per Inch)

No.	Mineral	Expansion[b] for the Following Percentages of Mineral Concentrate in Aggregate											
		2		5		10		25		50		100	
		H	L	H	L	H	L	H	L	H	L	H	L
1	Quartz plagioclase	...	...	357	171	318	171	338	168	362	189	425	236
2	Potash feldspar	388	160	362	212	331	231	369[c]	150	92	−192	95	−75
3	Biotite	370	129	294	176	325	126	265	210	480	344	506	368
4	Hornblende	355	213	370	323[d]	399	170	370	189	352	124	...	...
5	Magnetite	331	74	248	163	457	226	349	247	...	...	...	...
6	Sphene	415	291	488	312	425	268	...	...	...	...	...	...
7	Fresh "granodiorite"	...	...	...	...	...	...	...	...	...	...	331	10
8	Altered "granodiorite"	...	...	...	...	...	...	...	...	286	87	320	165
9	Siliceous magnesian limestone	...	...	...	...	2,430[e]	152[e]	...	...	...	...	...	...
10	Stellerite[f]	356	185	...	...	...	...	305[g]	237[g]	...	...	...	...

[a] Volume-change observations were made, after two years, on three cement-aggregate test bars for each condition. The mortar bars (1 in. by 1 in. by 6 in.; mix, 1 : 2 by weight; and water-cement ratio, 0.57 by weight) were sealed and stored over water at 100° F. [b] The abbreviations H and L denote, respectively, high-alkali cement, specially manufactured, containing 0.89% sodium oxide and 0.43% potassium oxide; and a commercial, low-alkali cement containing 0.18% total alkali as Na_2O. [c] Average of six bars. [d] Average of two bars only. [e] 20% of siliceous magnesian limestone in each case. [f] Age 270 days. [g] 33% stellerite.

same cements with an aggregate made from altered or weathered granodiorite and also with siliceous magnesium limestone of proved reactivity obtained from T. E. Stanton, M. Am. Soc. C. E., of the California Highway Commission.

While making the mineral separations it was found that occasional pieces of slightly altered granodiorite contained a zeolitic mineral known as stellerite. Since a zeolite reacts with the ordinary tap water used in washing and may be

eliminated by the washing operations, specimens were hand picked for high stellerite content and crushed and concentrated in the dry to obtain a sand containing 33% stellerite. Two series of test bars were made, one containing 2% stellerite and the other 33% stellerite, each mixed with the required complement of crushed granodiorite or quartz (see item 10, Table 8).

To date (1945) the test results are largely negative but they are of value nevertheless. These results may be summarized as follows:

(1) None of the bars showed clearly harmful expansion and presumably, therefore, the aggregate minerals are nonreactive with high-alkali cement;

(2) All the mortars made with high-alkali cement expanded more than those made with low-alkali cement; and

(3) Specimens containing large percentages of biotite mica show rather large expansions for mortar bars, but the rate of change over the last year has been so slow that these specimens cannot be placed definitely in the dangerous class.

The authors have assisted the writer materially in the work done under the fellowship and in searching for the causes of other concrete troubles, and this opportunity is taken to express appreciation for material assistance, advice, and free exchange of progress information.

Bibliography.—

(1) **"Repairs to Buck Power House and Dam,"** by Philip Sporn and H. A. Kammer, *Civil Engineering*, July, 1944, p. 285.

(2) **"Investigation of Causes of Delayed Expansion of Concrete in Buck Hydroelectric Plant,"** by H. A. Kammer and R. W. Carlson, *Proceedings*, A. C. I., Vol. 37, 1941, p. 665.

(4) **"Expansion of Concrete Through Reaction Between Cement and Aggregate,"** by Thomas E. Stanton, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 54. (*a*) p. 82. (*b*) Fig. 4, p. 63. (*c*) Table 5, p. 84.

(9) *The Delaware Water Supply News*, New York Board of Water Supply, Vols. 1 and 2, Nos. 1 to 28 and 34, 1938 and 1939.

(10) *Proceedings*, A.C.I., Vol. 38, 1942, p. 236-7.

(11) **"The Data of Geochemistry,"** by Frank Wigglesworth Clarke, *Bulletin No. 770*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1924, p. 458.

(12) *Proceedings*, A.C.I., Vol. 38, 1942, p. 236-28.

(13) **"Radium Content of Lavas from Lassen Volcanic National Park,"** by R. D. Evans and Howel Williams, *American Journal of Science*, May, 1935, pp. 441–452.

(14) **"Studies to Develop an Accelerated Test Procedure for the Detection of Adversely Reactive Cement-Aggregate Combinations,"** by Thomas E. Stanton, *Proceedings*, A.S.T.M., Vol. 43, 1943, Figs. 3 and 4, pp. 886–889.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

Discussion

By Leon Beskin, Alexander Dodge, and William C. Spiker

Leon Beskin,[15] Assoc. M. Am. Soc. C. E.[15a]—The structural concepts reviewed by the author cover the efforts of engineers, over a period of seventy years, to simplify the analysis of frameworks. By restricting the problem to that of a framework with no joint displacement, the unknowns are limited to joint rotations and to beam-end moments. Two expressions can be written defining the relation between the moments and the rotations at the two ends of each beam of a framework. The analysis of the framework is equivalent to the solution of the system of simultaneous equations represented by these expressions. All the so-called "methods of analysis" are only various procedures for the solution of the system of simultaneous equations. This explains why there is so little fundamental difference between all existent methods, and why the coefficients introduced by one writer can be reduced so easily to those derived by another writer. Practically every stress analyst has computed such relationships at one time or another, and it is improbable that, in this paper, any new physical interpretation is given to structural terms. The relationship between fixed points, carry-over factors, and stiffness factors, known in principle since the fundamental works of O. Mohr and Maurice Lévy,[11] can be found in various forms in more recent papers.[16]

The concept of node (fixed point) seems to have been introduced by Mr. Mohr for the special case of continuous beams. Later, it was extended to frameworks. A common practice—especially in Europe—is the analysis of frameworks by determining the locations of the fixed points.[17,12] For instance,

Note.—This paper by Camillo Weiss was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1945, by Ralph W. Stewart, and R. C. Brumfield; and June, 1945, by Max W. Strauss.

[15] Senior Stress Analyst, Eng. Dept., Consolidated Vultee Aircraft Corp., Nashville, Tenn.

[15a] Received by the Secretary May 29, 1945.

[11] "La Statique Graphique et Ses Applications Aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 1886–1888.

[16] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[17] "Analysis of the General Two-Dimensional Framework," by Yves Nubar, *Proceedings*, Am. Soc. C. E., April, 1944, p. 429.

[12] "Moving Loads on Restrained Beams and Frames," by R. C. Brumfield, *ibid.*, May, 1945, p. 627.

Professor Campus has taught these methods at Liége University in Liége, Belgium, for many years. These facts challenge the remark in the "Introduction" that "Usually such points are thought of in connection with beams * * *." The author's definitions of the fixed points are misleading. It is incorrect, for example, to state (see "Introduction") that "their position on the elastic curve is stationary," and (see heading, "Fixed Points and Carry-Over Factors") that: "When one end of a restrained member is rotated back and forth, the wave-like elastic curves will pass through a certain node." The amplitude of displacement of a node is different from zero, and, characteristically, a node is a point of inflection. From the principle of superposition, this point is fixed when there is only one applied end moment.

The relation between intercept ratios is well known in the form of an expression for carry-over factors. For continuous beams it is:

$$f_{BC} = \frac{K_{BC}}{2\,(K_{BA} + K_{BC}) - f_{AB}\,K_{BC}} \qquad (40a)$$

or

$$\frac{K_{BA}}{2 - f_{AD}} + \frac{K_{BC}}{2 - \dfrac{1}{f_{BC}}} = 0 \qquad (40b)$$

The corresponding relation for frameworks is also well known.[17,18] The writer, for example, has shown[18] that a simple derivation of the formula can be established, and that the formula can be written in a symmetric form:

$$\sum_X K_{BX} \frac{1}{2 - f_{BX}} = 0 \qquad (41a)$$

or, using the author's notation and defining all the intercept ratios from the common joint:

$$\Sigma K_{BX} \frac{n_{BX}}{3\,n_{BX} - 1} = 0 \qquad (41b)$$

In general, the use of carry-over factors leads to simpler formulas than those written in terms of nodal intercept ratios. For instance (compare Table 1):

$$d_{AB} = \frac{2 - f_{AB}}{2 + f_{BA}\,(1 - f_{AB})} \qquad (42a)$$

$$Q_{AB} = 1 - d_{AB} \qquad (42b)$$

$$F_{AB} = \frac{3\,f_{BA}}{2 - f_{BA}} \qquad (42c)$$

$$r_{AB} = \frac{6\,K_{AB}}{2 - f_{AB}} \qquad (42d)$$

and

$$n_{AB} = \frac{1}{1 + f_{AB}} \qquad (42e)$$

---

[18] *Proceedings*, Am. Soc. C. E., June, 1944, p. 913.

In the light of these remarks, it is fair to state, contrary to the author, that the so-called "Cross method" does not permit the analysis of frameworks from a different or novel point of view. The basic relations between end moments and end rotations can be established, either by geometric considerations based on the relation between moment and curvature, or by strain energy methods. All methods have the same basis, and, although the wording is different, the mathematical process is the same. For instance, consider the formula giving the end rotation of a beam as a function of the moments in the beam:

$$\theta_{AB} = \int_A^B \frac{(L - x)\, M}{E\, I}\, dx \qquad (43)$$

From geometric considerations, it appears that the curvature $\frac{M}{E\,I}$ produces a rotation $\frac{M\,dx}{E\,I} = d\phi$ between the two ends of an infinitesimal element dx, so that the deflection at one joint B, in reference to the tangent at the other joint A, is:

$$\delta_B = \int_A^B (L - x)\, d\phi \qquad (44)$$

If the reference line is rotated by an angle θ_{AB}, equal to the rotation at point A, the deflection at point B is equal to zero, which defines θ_{AB}:

$$\theta_{AB} = \frac{\delta_B}{L} \qquad (45a)$$

By application of strain energy methods, a dummy couple equal to 1 is placed at A, which produces moments $m = (L - x)/L$ at each point of the beam, and the rotation is:

$$\theta_{AB} = \int_A^B m\, d\phi \qquad (45b)$$

Eqs. 45*a* and 45*b* lead to the same relation (Eq. 43). In the "Introduction," Mr. Weiss states:

> "Whereas the method of least work is based on the concepts of work and energy * * * the Cross method may be considered as based on a complex of concepts that can be summed up under the term 'restraint.'"

This statement ignores the fact that Eq. 43 is the starting point for the Cross method as well as for other methods. Continuing, the author states: "Work and deformation imply the action of a force. Restraint, however, is entirely a function of the structure itself." This seems to imply that the concept of restraint can be defined without introducing into it the concept of force. Paradoxically, the author defines restraint thus (see heading, "Stiffness Factors and Distribution Factors"): "In general the resistance to rotation * * * will be E times a stiffness factor or restraint factor '*r*.'" In other words, far from being exclusively a function of the structure, restraint is nothing but the ratio

of an externally applied load (generally a couple) to a displacement (generally a rotation), multiplied by E.

The author's opinion (see "Introduction") that "The method of moment distribution * * * is as good a method as can be devised," accepted without question by many structural engineers, should nevertheless be challenged. The following summary is intended to clarify the comparison between various methods.

Methods of solving simultaneous equations can be divided into three main groups—succession elimination of unknowns, determinants, and successive approximations. Determinants require an excessive amount of computations when there are more than three or four unknowns, and this method is of little practical interest. The best solution by successive elimination was that presented by Karl F. Gauss, and it has been applied to the problem of continuous beams as the method of conjugate points.[11,16] The relationship between these two methods is easy to establish. To make the problem more general, beams of variable sections are considered. Numbering the supports by even numbers $2j$, and the spans by odd numbers $2j + 1$, the three-moment equation is:

$$B_{2j-1} M_{2j-2} + C_{2j} M_{2j} + B_{2j+1} M_{2j+2} = H_{2j} \qquad (46)$$

The quantities B, C, and H are defined by the following relations, in which x is the abscissa in each span, measured in the direction of increasing j; ζ is the ratio x/L, and m is the statical moment in each span:

$$B_{2j-1} = L_{2j-1} \int_0^1 \frac{\zeta (1 - \zeta) \, d\zeta}{I_{2j-1}} \qquad (47a)$$

$$C_{2j} = L_{2j-1} \int_0^1 \frac{\zeta^2 \, d\zeta}{I_{2j-1}} + L_{2j+1} \int_0^1 \frac{(1 - \zeta)^2 \, d\zeta}{I_{2j+1}} \qquad (47b)$$

and

$$H_{2j} = \int_0^1 \frac{\zeta \, m_{2j-1} \, d\zeta}{I_{2j-1}} + \int_0^1 \frac{(1 - \zeta) \, m_{2j+1} \, d\zeta}{I_{2j+1}} \qquad (47c)$$

Eliminating unknowns, successively, to the moment M_{2j-4}, inclusive, Eq. 46 (written for the support $2j - 2$) is transformed into:

$$C'_{2j-2} M_{2j-2} + B_{2j-1} M_{2j} = H'_{2j-2} \qquad (48a)$$

The moment M_{2j-2} is eliminated by the linear combination of Eqs. 46 and 48a and leads to an equation similar to Eq. 48a:

$$C'_{2j} M_{2j} + B_{2j+1} M_{2j+2} = H'_{2j} \qquad (48b)$$

which shows that quantities C'_{2j} and H'_{2j} are defined by the following relations of recurrence:

$$C'_{2j} = C_{2j} - \frac{B^2_{2j-1}}{C'_{2j-2}} \qquad (48c)$$

and

$$H'_{2j} = H_{2j} - \frac{B_{2j-1}}{C'_{2j-2}} H'_{2j-2} \qquad (48d)$$

The first member of Eq. 48*b* defines a linear relation between M_{2j} and M_{2j+2}, which shows that the carry-over factor f_{2j+1} is:

$$f_{2j+1} = \frac{B_{2j+1}}{C'_{2j}} = \frac{B_{2j+1}}{C_{2j} - f_{2j-1}\,B_{2j-1}} \qquad (49)$$

Eq. 49 can be identified easily with Eq. 40*a*, since, in the case of beams with constant sections, the following relations hold:

$$B_{2j+1} = \frac{1}{6\,K_{2j+1}} \qquad (50a)$$

and

$$C_{2j} = \frac{1}{3\,K_{2j-1}} + \frac{1}{3\,K_{2j+1}} \qquad (50b)$$

Eq. 48*b* can be written for all the spans, and the equation for the last span contains only one unknown moment, which is thus directly determined. Using Eq. 48*b* in reversed sequence, all the moments are defined, successively, by relations such as:

$$M_{2j} = -\frac{B_{2j+1}}{C'_{2j}}\,M_{2j+2} + \frac{H'_{2j}}{C'_{2j}} \qquad (51)$$

Thus, the method of successive eliminations leads to the use of Eqs. 48*c*, 48*d*, and 51, representing eight numerical operations per span, three being independent of the loading conditions (determination of the coefficients C'_{2j}).

The Cross method requires five numerical operations per span, per cycle—two distributions, two carry-over operations, and one summation. When the cycles are completed, an over-all addition, equivalent to at least one operation per cycle, is required. Thus, there are six operations per cycle, instead of a total of eight operations by the direct method. If checks are made, it can be shown that the direct method requires thirteen operations per span (additional calculation of coefficients C' and H' from the second end of the beam), whereas the Cross method is equivalent to nine operations per span, per cycle. The preliminary operations are equivalent in both methods. For instance, the two carry-over factors in a span are $\frac{B_{2j-1}}{C_{2j-2}}$ and $\frac{B_{2j-1}}{C_{2j}}$; and the determination of the factors H is equivalent to the determination of the fixed-end moments. Thus, as two or three cycles, and very often more, are required by the Cross method, it is more expeditious, in the case of beams, to apply the direct method. In addition, three operations per span (or six operations, including checks) are independent of the loading in the direct method. Thus, for additional loading conditions, seven operations per span (including checks) are required by the direct method, whereas nine operations per span, per cycle, are required by the Cross method. In this case, the Cross method is definitely inefficient.

The direct method can be used in the case of frameworks. However, because there is no initial equation of the type of Eq. 48*a* along a closed circuit, the direct method is generally transformed into a method of successive approximations. In the initial form, complications increase with the number of parallel beams (stories) or columns (spans). For instance, for four parallel

rows (spans or stories), the elimination should be made in the sequence (see Fig. 7): 1, 2, 5, 3, 6, 9, 4, 7, 10, 13, 8, 11, 14, 17, $\cdots$. In the case of two rows of joints, and especially when the joints are not fixed (Vierendeel beams such as high buildings with lateral load), the direct method is more advantageous

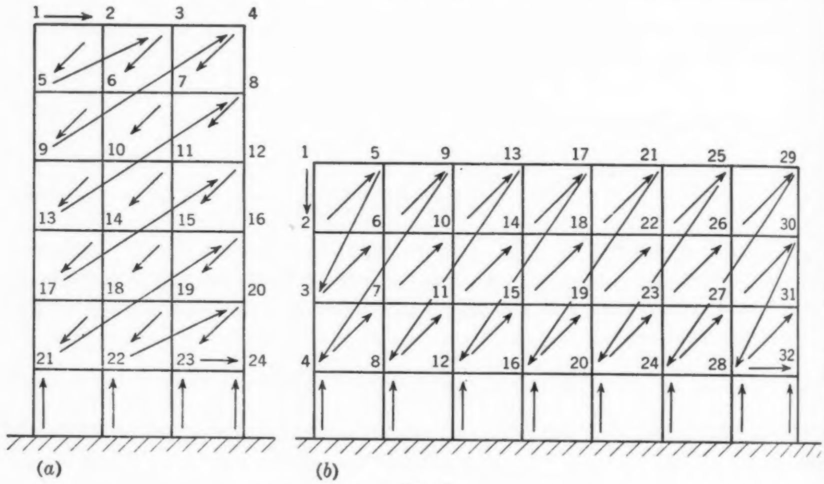


FIG. 7

than methods of successive approximation, but this advantage disappears for a greater number of rows of joints. Finally, contrary to general opinion, the accuracy of the direct method does not decrease when there are many spans, because the coefficients, such as $\frac{B^2{}_{2j-1}}{C'{}_{2j-2}}$ (see Eq. 48c), are of the order of magnitude of one quarter of the coefficients C_{2j} so that small errors tend to disappear, instead of accumulating. This fact is precisely the cause of the rapid convergence of methods of successive approximations.

Methods of successive approximations have been developed by physicists and mathematicians, in connection with various problems in physics. The first systematic work in this field can be attributed to J. Fourier,[19] who developed his theories around 1820. However, it was not until between 1900 and 1910 that various mathematicians, principally J. Fredholm and D. Hilbert,[20] established the theory in its present form. Their work dealt mainly with equations containing an infinite number of unknowns, because physical problems generally lead to such equations. Thus, the theories are greatly simplified when applied to structural problems with a finite number of unknowns.

Methods of successive approximations can be presented as follows (see Eqs. 47). A system of linear equations can be written in the form:

$$\sum_j X_j\, a_{ij} = Y_i \quad \text{.......................} (52)$$

[19] "Théorie Analytique de la Chaleur," by J. Fourier, Didot, Paris, France, 1822.

[20] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 1931.

in which X_j represents the unknowns, a_{ij} the coefficients (defined by the structure only), and Y_i the second members (defined by the structure and the loading). The matrix form of Eq. 52 is:

$$|X_j|\,|a_{ij}| = |Y_i| \quad \text{(53a)}$$

and the solution is:

$$|X_j| = |Y_i|\,|a'_{ij}| \quad \text{(53b)}$$

in which the matrix $|a'_{ij}|$ is the reciprocal of the matrix $|a_{ij}|$; thus:

$$|a'_{ij}| = \frac{1}{|a_{ij}|} = |a_{ij}|^{-1} \quad \text{(54a)}$$

Eq. 54*a* has no significance until the numerical computation which it represents is defined. For this purpose, the matrix $|a_{ij}|$ is written as:

$$|a_{ij}| = |a_{ii}|\left|\frac{a_{ij}}{a_{ii}}\right| = |a_{ii}|[|E| + |b_{ij}|] \quad \text{(54b)}$$

The first equality in Eq. 54*b* signifies that each column of the square matrix $|a_{ij}|$ is divided by its diagonal term a_{ii} and the resulting matrix $\left|\frac{a_{ij}}{a_{ii}}\right|$ can be multiplied by the column matrix $|a_{ii}|$ to find the matrix $|a_{ij}|$. In $\left|\frac{a_{ij}}{a_{ii}}\right|$, the diagonal terms $\frac{a_{ii}}{a_{ii}}$ are equal to unity. The second equality in Eq. 54*b* signifies that $\left|\frac{a_{ij}}{a_{ii}}\right|$ can be written in the form of a sum of the identical matrix $|E| = |e_{ij}|$ (with: $e_{ii} = 1$ and $e_{ij} = 0$), and of the matrix $|b_{ij}|$ $\left(\text{with: } b_{ii} = 0 \text{ and } b_{ij} = \frac{a_{ij}}{a_{ii}}\right)$. It can be shown that the relation—

$$(1 + r)^{-1} = \frac{1}{1 + r} = 1 - r + r^2 - r^3 \cdots \pm r^n \cdots \quad \text{(55)}$$

—holds for the matrix $|E| + |b_{ij}|$; thus:

$$(|E| + |b_{ij}|)^{-1} = |E| - |b_{ij}| + |b_{ij}|^2 \cdots \pm |b_{ij}|^n \cdots \quad \text{(56)}$$

The conditions of convergence of the series in the right-hand member of Eq. 56 are satisfied in the case of equations corresponding to redundant structures. Thus, Eq. 53*b* can be written in the form:

$$|X_j| = |Y_i|\,|a_{ij}|^{-1} = \left|\frac{Y_i}{a_{ii}}\right|(|E| - |b_{ij}| \cdots \pm |b_{ij}|^n \cdots) \quad \text{(57)}$$

Eq. 57 has a definite significance, since multiplication and addition of matrices are defined operations. This equation is the basis of the "method of relaxation" developed by R. V. Southwell,[21] of which the Cross and the Maney-

[21] "Systematic Relaxation of Constraints," by R. V. Southwell, *Proceedings*, Royal Soc. of London, Vol. CLI, Series A, 1935, pp. 56–95; Vol. CLIII, Series A, 1935–1936, pp. 41–76.

Goldberg[22,23,24,25] methods are only special applications. In the case of frameworks, the latter method can be reconciled with the method of relaxation as follows:

In the Cross method, quantities that can be called "total unbalanced moments" (M_j) are the principal unknowns. These quantities are related to the fixed-end moments $M^o_{j,i}$ (moment at joint j, in beam ji), by

$$M_j + \sum_i M_i \, b_{i,j} = \sum_i M^o_{j,i} = M^o_j \qquad (58)$$

Factors $b_{i,j}$ are the products of distribution factors by carry-over factors:

$$b_{i,j} = d_{i,j} f_{i,j} \qquad (59)$$

The final-end moments $M_{i,j}$ are defined by:

$$M_{i,j} = M^o_{i,j} - M_i \, d_{i,j} - M_j \, b_{j,i} \qquad (60)$$

The method of successive approximation can be expressed by the following relations between successive unbalanced moments:

$$M^1_j = - \sum_i b_{i,j} \, M^o_i \qquad (61a)$$

and

$$M^{n+1}_j = - \sum_i b_{i,j} \, M^n_i \qquad (61b)$$

so that:

$$M_j = \sum_{n=1}^{\infty} M^n_j \qquad (62)$$

In the Cross method, Eqs. 61 are written in the following detailed form: Distribution—

$$\Delta M^{n+1}_{i,j} = - M^n_i \, d_{i,j} \qquad (63a)$$

carry over—

$$\partial M^{n+1}_{j,i} = \Delta M^{n+1}_{i,j} f_{i,j} \qquad (63b)$$

unbalance—

$$M^{n+1}_j = \sum_i \partial M^{n+1}_{j,i} \qquad (63c)$$

Eq. 60 is written in the form:

$$M_{i,j} = M^o_{i,j} + \sum_{n=1}^{\infty} \Delta M^n_{i,j} + \sum_{n=1}^{\infty} \partial M^n_{i,j} \qquad (64)$$

The method has been perfected by elimination of Eq. 63a, so that:

$$\partial M^{n+1}_{j,i} = - M^n_i \, b_{i,j} \qquad (65)$$

[22] "Wind Stresses in the Steel Frames of Office Buildings," by W. M. Wilson and G. A. Maney, *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Urbana, 1915.

[23] "Vertical-Load Analysis of Rigid Building Frames Made Practicable," by John E. Goldberg, *Engineering News-Record*, November 12, 1931, pp. 770–772.

[24] "Simplified Methods for the Analysis of Multiple Joint Rigid Frames," by G. A. Maney and John E. Goldberg, *Bulletin*, Northwestern Univ., School of Eng., 1932.

[25] "Wind Stresses by Slope Deflection and Converging Approximations," by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–985.

whereas Eq. 64 is written as:

$$M_{i,j} = M^o{}_{i,j} - M_i\, d_{i,j} + \sum_{n=1}^{\infty} \partial M^n{}_{i,j} \qquad (66)$$

The method thus developed is easily identified with Eq. 57, written as:

$$|M_j| = |M^o{}_i| \left(|E| - |b_{i,j}| + \cdots \pm |b_{i,j}|^n + \cdots\right) \qquad (67a)$$

$$|M_j| = |M^o{}_j| - |M^o{}_i|\,|b_{i,j}| + \cdots \pm |M^o{}_i|\,|b_{i,j}|^n + \cdots \qquad (67b)$$

and

$$|M_j| = |M^o{}_j| + |M^1{}_j| + \cdots + |M^n{}_j| + \cdots \qquad (67c)$$

Eq. 67*c* is the result of the repeated substitution of Eq. 61*b*, written in the form—

$$|M^{n+1}{}_j| = -\,|M^n{}_i|\,|b_{i,j}| \qquad (68)$$

—in Eq. 67*b*.

Thus, it appears that the Cross method is an application, to structural problems, of a conventional method of mathematical analysis. The slope-deflection method, developed by G. A. Maney, M. Am. Soc. C. E., and John E. Goldberg,[22,23,24,25] Assoc. M. Am. Soc. C. E., is equivalent in its initial form to the improved Cross method (elimination of Eq. 63*a*). The difference is in the choice of unknowns which are the slope deflections instead of the "total unbalanced moments." The final moments are defined by a relation similar to Eq. 60, in which $r_{i,j}$ is a restraint factor:

$$M_{i,j} = M^o{}_{i,j} - E\,\theta_i\, r_{i,j} - E\,\theta_j\, r_{j,i}\, f_{j,i} \qquad (69)$$

From the relation

$$d_{i,j} = \frac{r_{i,j}}{\sum_j r_{i,j}} \qquad (70)$$

it is found that

$$M_i = E\,\theta_i \sum_j r_{i,j} \qquad (71)$$

which shows the relationship between the two methods. The Maney-Goldberg method appears to be the simpler, since it makes use of the elementary concept of joint rotation instead of the complex concept of total unbalanced moment which, in final analysis, is the external couple required to produce the actual joint rotation when the other joints are fixed and when no other load is applied.

The original Maney-Goldberg method has been improved by its authors in the following manner: Dividing the joints into two groups i and j defined in such a manner that every span connects two joints belonging to the two groups, Eq. 67*a* can be written in the form—

$$|M_j| = \left(|M^o{}_j| - |M^o{}_i|\,|b_{ij}|\right)\left(|E| + |b_{i,j}|^2 + \cdots + |b_{i,j}|^{2n} \cdots\right) \qquad (72a)$$

and

$$|M_i| = |M^o{}_i| + \left(|M^o{}_j| - |M^o{}_i|\,|b_{i,j}|\right) \times |b_{i,j}| \left(|E| + \cdots + |b_{i,j}|^{2n} \cdots\right) \qquad (72b)$$

Since the sum $|M^o{}_j| - |M^o{}_i||b_{i,j}|$ forms one matrix, the number of operations required by Eqs. 72 is half of the number necessitated by Eq. 67*a*, to obtain the same approximation. This short cut is essential when the convergence is slow (as, for example, for structures with sidesway).

The Maney-Goldberg method may appear longer than the Cross method, because it necessitates the computation of moments equivalent to rotations. However, since a check of the results is indispensable, it is necessary to calculate the rotations when the Cross method is applied, to find whether the relations between rotations and end moments are satisfied by the solution. The additional operation is included in the Maney-Goldberg method which is self-checking. When this operation is added to the Cross method, the latter becomes self-checking, and the number of operations is the same by both methods, provided the Maney-Goldberg short cuts are applied to the Cross method.

Another method of successive approximations results from a different interpretation of Eq. 67*a*: If the matrix,

$$|B_{i,j}| = |E| - |b_{i,j}| + |b_{i,j}|^2 \cdots \pm |b_{i,j}|^n \cdots\cdots\cdots \quad (73)$$

is determined, the total unbalanced moments are defined simply by:

$$|M_j| = |M^o{}_i||B_{i,j}| \cdots\cdots\cdots \quad (74)$$

This method can be applied to slope deflection as well. The numerical calculation of the matrix $|B_{i,j}|$ is about as complex as the determination of one actual solution by Eq. 67*c*. On the other hand, this operation is done only once for a given structure, after which Eq. 74 defines all the total unbalanced moments with a minimum of operations for any loading condition, avoiding the tedious repetition of successive approximations in each case. This method can easily be identified with the method of conjugate points, since, in both methods, the moments at any joint are defined directly in terms of a linear combination of the fixed-end moments at all the joints.

In conclusion, the direct method of successive elimination is always the most advantageous in the case of one row of spans, and it is generally the most advantageous for two rows of spans. In all the other cases, methods of successive approximations are more powerful. When only one condition or very few loading conditions are examined, the Maney-Goldberg method is the most expeditious, although the Cross method can be improved to obtain the same efficiency. When many loading conditions are examined, the method of successive elimination, transformed into a method of successive approximations (conjugate points in frameworks), is generally the best. The section under the heading, "General Principles of Standardization," in Appendix II, will help to avoid the use of such cumbersome notations as FEM for fixed-end moments and $\overline{\text{DL}}$ and $\overline{\text{LL}}$ for dead load and live load.

ALEXANDER DODGE,[26] ASSOC. M. AM. SOC. C. E.[26a]—The slope-deflection method has been used in this paper to derive equations for members of constant moment of inertia. In appraising the work done by Mr. Weiss, it will be

[26] Engr., U. S. Engrs., Portland, Ore.

[26a] Received by the Secretary May 31, 1945.

instructive to compare it with parallel studies by the writer[27] in 1939, which involved the development of the moment-area principle for members of varying moments of inertia. The writer's studies included the effect of sidesway and the construction of influence lines. The definition of fixity constitutes the main difference in the papers. In the writer's concept, the "end fixity" is defined as the ratio of $\frac{Z'}{Z}$, in which Z' is the exact moment at point B due to unit moment at point A, and Z is the moment at point B due to a unit moment at point A if end B is fixed. For members with a constant moment of inertia, $Z = 0.5$. Referring to Fig. 2, the "end fixity" is

$$f_{ba} = \frac{\Sigma S_p}{\Sigma S_p + S_{ba}} \qquad (75a)$$

in which S_p is the exact stiffness corresponding with the stiffness factor r in the author's concept, or

$$f_{ba} = \frac{R_{BA}}{R_{BA} + 3\,K_{BA}} \qquad (75b)$$

Eq. 75*b* will be recognized as a part of Eq. 16. The reversed subscript for the symbol K denotes that the stiffness at end B of member AB is used, which is an important distinction in analyzing a member of varying moment of inertia. The carry-over factor may be looked upon as the exact moment at joint B due to unit moment at end A. With these changes in the subscript notations as explained in Appendix I, Eq. 16 would be changed to

$$f_{BA} = \tfrac{1}{2}\,\frac{R_{BA}}{R_{BA} + 3\,K_{BA}} \qquad (76)$$

thus simplifying most of the equations derived from it. In itself, this is not an important change, but the fact that all terms in Eq. 76 refer to the same point (joint B) will certainly be appreciated by engineers who may have practical use for the concepts. The definition of end fixity expressed by Eqs. 75 has several advantages: (1) It is a linear function of the carry-over factor and, therefore, has a physical significance that can be visualized easily and clearly; and (2) it makes the expression for the exact stiffness of the member, S_p, very simple, namely:[28]

$$S_p = \frac{C'}{C}\,S \qquad (77)$$

in which S is the stiffness of the member at joint B if end A is hinged; C is the ratio of stiffness S to the stiffness of the member at joint B if end A is fixed; and[28]

$$C' = \frac{C}{1 - f\,(1 - C)} \qquad (78)$$

Charts for the constants S and C have been prepared and published by

[27] "Solution of Rigid Frames by Coefficients of Unit Moment," by Alexander Dodge, thesis presented to the University of Washington, Seattle, Wash., in 1939 in partial fulfilment of the requirements for the professional degree of Civil Engineer.

[28] "Design of Continuous Frames Having Variable Moment of Inertia," by Thor Germundsson, *Civil Engineering*, October, 1932, p. 647.

Thorbjörn Germundsson,[28] Assoc. M. Am. Soc. C. E. Values of C' for any variation of moment of inertia can be selected from Fig. 8. For a constant moment of inertia, Eq. 77 becomes:

$$S_p = \frac{C'}{0.75} \times \frac{3\,I}{L} = \frac{4\,C'\,I}{L} \qquad (79)$$

In general, referring to Eq. 23, the end moment at end A is not equal to $Q_{AB}\,C_{AB}$; but, by Eq. 16, it is equal to $\frac{R_{AB}}{R_{AB} + 3\,K_{AB}}\,C_{AB}$, which is a constant

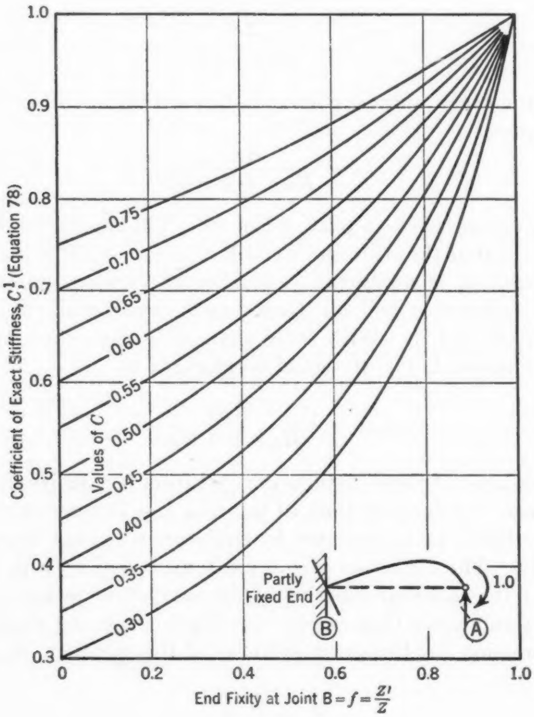


Fig. 8

that is independent of the characteristics of joint B. In Eq. 17*a*, r_{AB} has a limit of $3\,K_{AB} \leqq r_{AB} \leqq 4\,K_{AB}$, which shows that r_{AB} is a function of the characteristics of joint B also. Therefore, using Fig. 1 as an example, it should be kept clearly in mind that, for this particular frame, the relation $Q_{XY} = 2 f_{YX}$ applies to no member except span BA, because $Q_{XY} \neq \frac{R_{XY}}{R_{XY} + 3\,K_{XY}}$.

In order to generalize the subject, the effect of varying moment of inertia should be examined. Using charts prepared by Mr. Germundsson,[28] with

Eqs. 16 and 17*a*, the values in Table 4 were computed for the ratio $b = \frac{I}{I'} = 0.08$, as applying to four designs of beam AB. The values common to all beams were assumed to be: $R_{AB} = 2.0$; the stiffness factor at end A of the member AB, with end B hinged, is equal to $S_{ab} = 1.0$, and $Q_{AB} = 67\%$ when $Q_{BA} = 0$. If the values of Table 4 were plotted on the chart in Table 1, they would be found scattered over a wide area. This indicates that, in general, no simple relation exists between values of Q and n. An important modification would also be noted in Eq. 22*b*, the constant of which would be changed from 2 to 1.333, 0.908, and 3.33, respectively, for the first three beams in Table 4.

TABLE 4.—CONSTANTS FOR BEAM AB AND VARIOUS COMBINATIONS OF STRAIGHT HAUNCHES

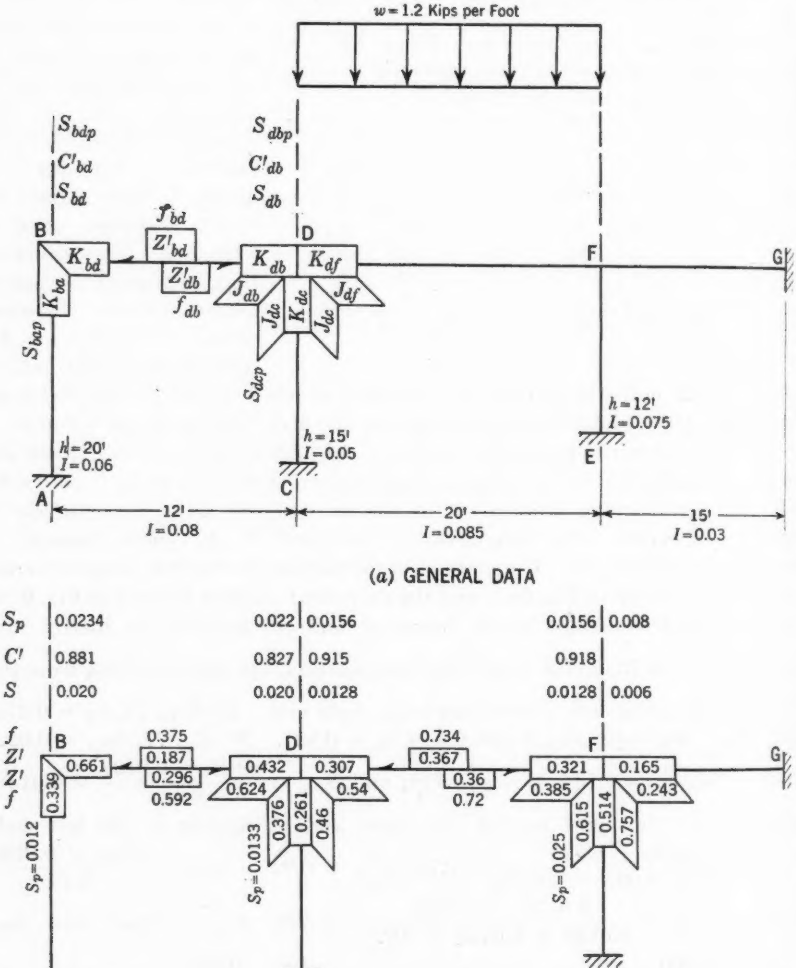
Haunches at:	Ratio, haunch to span length[a]	f_{BA}	n_{BA}	Q_{AB} (end B fixed)[b]
A and B	0.4	0.5	0.667	0.462
End A	0.8	0.734	0.572	0.568
End B	0.8	0.2	0.833	0.568
Uniform section	....	0.333	0.75	0.6

[a] The ratio of minimum to maximum moment of inertia is 0.08 in each case. [b] The value of Q_{AB} with end B hinged is 0.67 in each case.

The basic structural concepts are expressed fully by Eqs. 13, 16, 75, and 77, and their application to the solution of various structures, including frames with sidesway and varying moment of inertia as well as to the construction of influence lines has been demonstrated previously.[28] A typical example is demonstrated in Fig. 9. The method of tabulating the needed values in terms of symbols is shown in Fig. 9(*a*), and the numerical solution is given in Fig. 9(*b*). Recalling that constants for a beam of uniform moment of inertia are: $S = \frac{3\,I}{L}$, $C = 0.75$, and $Z = 0.5$, the computations are made starting from the left end of the frame and proceeding to the right end. By Eqs. 75, $f_{bd} = 0.375$. With this value enter Fig. 8 and read $C'_{db} = 0.827$. By Eq. 77, $S_{dbp} = 0.022$; by Eqs. 75, $f_{df} = 0.734$; $C'_{fd} = 0.918$; and $S_{fdp} = \frac{0.918 \times 0.0128}{0.75} = 0.0156$. Beginning at the right end of the frame and proceeding to the left end: $f_{fd} = \frac{0.008 + 0.025}{0.008 + 0.025 + 0.0128} = 0.72$; $C'_{df} = 0.915$; $S_{dfp} = \frac{0.915 \times 0.0128}{0.75} = 0.0156$; $f_{db} = \frac{0.0156 + 0.0133}{0.0156 + 0.0133 + 0.02} = 0.592$; $C'_{bd} = 0.881$; and $S_{bdp} = \frac{0.02 \times 0.881}{0.75} = 0.0234$. By Eq. 13, $K_{ba} = \frac{0.012}{0.012 + 0.0234} = 0.339$. It should be noted that factor K_{ba} corresponds to factor d_{BA} in the author's concept.

All other K-values are computed similarly. In practice, the computations are made on a slide rule and recorded directly in Fig. 9(*a*). It will be noted that, in addition to the foregoing computations, the coefficients J are also recorded in Fig. 9. These coefficients indicate the proportion in which the coefficient Z' received at the joint is divided between the remaining members;

for example, $J_{db} = \dfrac{K_{db}}{K_{db} + K_{dc}} = \dfrac{0.432}{0.432 + 0.261} = 0.624$, etc. Noting that the coefficients K and J always carry a minus sign and that coefficients Z' have a



(*b*) COMPUTATIONS

FIG. 9.—SOLUTION OF A RIGID FRAME

plus sign, and assuming clockwise moments to be positive, for the loading conditions of Fig. 9(*a*), the final moments will be as follows: Fixed-end moments $= \dfrac{w\,L^2}{12} = \dfrac{1.2 \times 20^2}{12} = 40$ kip-ft; $M_{DF} = -\,40\,(1 - K_{df}) + 40\,(-\,K_{fd} \times Z'_{df})$

$= -32.43$ kip-ft; $M_{DB} = M_{DF}\,J_{db} = 20.24$; $M_{BD} = M_{DB}\,Z' = 3.78$; $M_{AB} = 3.78 \times -1 \times 0.5 = -1.89$; $M_{DC} = -32.43 \times -0.376 = 12.19$; $M_{CD} = 12.19 \times 0.5 = 6.10$; $M_{FD} = 40\,(1 - 0.321) + (-40 \times -0.307 \times 0.36) = 31.58$; $M_{FE} = 31.58 \times -0.757 = -23.91$; $M_{EF} = -23.91 \times 0.5 = -11.96$; $M_{FG} = 31.58 \times -0.243 = -7.67$; and $M_{GF} = -7.67 \times 0.5 = -3.84$. If a frame is built up of members with varying moments of inertia, the same procedure is used, except that the coefficient C' would be selected from Fig. 8 to correspond with the respective beam C-values.

The author's paper, being essentially mathematical, might be improved if its definitions and symbols were adapted to a more general case. For example, for the third beam in Table 4, the node caused by end rotation is closer to the generating end A if end B is fixed. Therefore, the definition of the carry-over factor as the ratio of the shorter nodal intercept to the longer nodal intercept would be in error. For these reasons, the definition of the "carry-over factor" as originally stated by Hardy Cross, M. Am. Soc. C. E., and so well rooted in engineering literature, should not be altered. There is an objection also to symbol r being used to designate the restraint or stiffness factor. The symbol r, designating the word "ratio," has been used to designate Professor Cross' "carry-over" factor, and tables have been prepared, published,[28] and broadly used in structural analyses. The symbol S, abbreviating a word stiffness, also has a good logical reason to be retained to represent the "stiffness" of a beam, and S_p "the exact stiffness," where the subscript p denotes "a partly fixed or partly restrained beam." As noted in Appendix II, the letter symbol $K = \frac{I}{L}$ equals the stiffness factor or a factor of proportionality of stiffness. Sometimes the letter K also denotes "the relative stiffness of a member" when compared with other members. Therefore, to avoid confusion, r_{BX} in Eq. 10 should not be referred to as "the stiffness factor," but as "the stiffness" or rather, "the exact stiffness." The meaning of "exact stiffness" may be clarified by referring to the "unit stiffness" concept which is defined thus: If a unit moment applied at the hinged end of a beam is causing unit rotation of the same end (for a given degree of restraint at the other end) the beam is said to have a "unit stiffness" at the end where the unit moment is applied. The "exact stiffness," then, is an inverse function of the true rotation angle at the end of the beam at which the unit moment is applied.

WILLIAM C. SPIKER,[29] M. AM. SOC. C. E.[29a]—A possible method is suggested by the author whereby engineers may eventually come nearer to agreement as to the simplest possible procedure for computing stresses in rigid frames. If engineers can agree on fundamental concepts and uniform symbols, progress will have been made.

The author and the writer agree that the "point of inflection" is an important and fundamental concept. Apparently they do not agree as to the importance of θ as a measure of resistance although they do agree as to the

---

[29] Cons. Engr., Atlanta, Ga.

[29a] Received by the Secretary June 4, 1945.

importance of resistance as a concept. The writer especially does not accept the author's symbols "*d*," "*r*," and "*n*" because they do not suggest what they mean and are in conflict with other symbols already well established.

Very strongly, the writer agrees with the author's idea of charting the location of points of inflection; but he is of the opinion that Table 1 is either not clear or is incorrect; and that the formulas of the paper are unnecessarily abstruse.

For computing tables and charts of θ-values, resistances, points of inflection, etc., the writer uses principles and formulas that can be deduced from the equations of the slope-deflection method and from the routine of its application. For convenience, the fixed-end moment FEM is taken equal to 100 or multiples of 100. Values of L are then equal to the square root of 1,200, when the unit load (w) is equal to 1, or multiples of 1.

Instead of using $I/L = K$ for all spans, that ratio is used only where one end of a member is actually fixed and where reference is made to the particular member along which "carry overs" are being made and for which equivalent K-values are being computed.

Reference can be made to the short cut or special case of the slope-deflection equation for beams hinged at one end; thus,

$$M_{A-B} = 3\,E\,K\,\theta_A - 3\,E\,K\,R - (\text{FEM})_1 \quad \text{(80)}$$

in which $(\text{FEM})_1$ is the fixed-end moment of the beam when only one end of the beam is fixed or assumed to be fixed. It is the algebraic sum of FEM and the carry over. In Eq. 80, the quantity $3\,E\,K\,\theta$ is actually $4\,E\,(\frac{3}{4}\,K)\,\theta$. In other words, K would be $\frac{3}{4}\,K$ if conditions remained the same except that the other end of the beam were fixed. This reduced K may be called the true, effective, or equivalent, value of K; but whatever it is called it designates a measurable, computable (or determinable) amount of resistance to the rotation of the end of a member. It is always less than the ratio I/L except when the other end of the member is actually fixed; or when the other end of the member is being rotated in the same direction as the end in question, and carry overs are omitted. The formula for computing the reduction of effectiveness of I/L at end of any member is taken from the slope-deflection method; thus:

$$K_{A-B}\ (\text{equivalent value}) = K_{A-B} - \left[\frac{(0.5\,K)^2}{K_{A-B} + \Sigma K_B\ (\text{effective values})}\right] \quad \text{(81)}$$

In Eq. 81, ΣK_B (effective values) is the K-value for members that are actually fixed at the other end, plus K_B (equivalent values) for members that are not fixed at the other end. The true, or effective value of K is always I/L or a percentage of that ratio.

The total carry over is always the product of $\dfrac{0.5\,K}{K_{A-B} + \Sigma K_B\ (\text{effective values})}$ and the algebraic sum of the fixed-end moment for beam AB at end B and the values of $(\text{FEM})_1$ for members meeting AB at end B.

The product $4\,\theta$ in the slope-deflection equation is always the algebraic sum of $(FEM)_1$'s divided by ΣK (effective values) at the joint. Then the application of the slope-deflection equation to the known values of θ determines the moments and checks them for continuity. Also, it detects an error in the location of the point of contraflexure in columns in case the points of contraflexure in the columns have been approximated.

Corrections for *Transactions:* In January, 1945, *Proceedings*, on page 65, the couple acting at joint A, Fig. 2, should be counterclockwise instead of clockwise as shown; and on page 69, in the denominator of Eq. 20*a*, change "f_{BA}" to "F_{BA}."

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

UTILIZATION OF GROUND-WATER STORAGE IN STREAM SYSTEM DEVELOPMENT

Discussion

By G. B. Drummond, Raphael G. Kazmann, Donald M. Baker, and Hyde Forbes

G. B. Drummond,[9] Assoc. M. Am. Soc. C. E.[9a]—People of the semi-dehydrated west read with interest anything about water, particularly a paper as able as that prepared by Mr. Conkling. However, it is to be remembered that the most comprehensive understanding of the phenomena of waters is of little avail unless administrative action conforms to correct scientific knowledge.

In New Mexico the laws covering ground water have been written on the basis of a scientific knowledge of hydrology. The State Constitution defines the doctrine of "appropriation" when it states:

> "The unappropriated water of every natural stream, perennial or torrential, within the State of New Mexico, is hereby declared to belong to the public and to be subject to appropriation for beneficial use, in accordance with the laws of the state. Priority of appropriation shall give the better right."

The Constitution continues: "Beneficial use shall be the basis, the measure and the limit of the right to the use of water."

The statute of the State of New Mexico passed in 1927 authorizing the appropriation of ground water was declared unconstitutional because of a technical error. In this case (Yeo vs. Tweedy, 286 Pacific 970), the New Mexico Supreme Court discussed in detail the common law principle that an owner in fee has dominion to the center of the earth, but declared that a sensible pursuit of that theory required reasonable use of any right and that the use shall be of such a nature as to avoid injury to others. On the basis of that decision, the present statute was passed in 1931. By this statute, "Waters

Note.—This paper by Harold Conkling was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1945, by Raymond A. Hill; April, 1945, by Clarence S. Jarvis, and John R. Charles; May, 1945, by C. W. Sopp, and J. A. Bradley; and June, 1945, by Irving B. Crosby.

[9] Asst. Prof., Civ. Eng., Univ. of New Mexico, Albuquerque, N. Mex.

[9a] Received by the Secretary May 22, 1945.

of underground streams, channels, artesian basins, reservoirs or lakes, having reasonably ascertainable boundaries" are public waters subject to appropriation for beneficial use.

The law is administered by the State Engineer to whom persons desiring to drill wells for irrigation or industrial purposes make application. After publication, if an interval passes during which no objections are filed and if the State Engineer ascertains that there are available unused waters in the area, a drilling permit is issued subject to the rights of previous appropriators from the source. If protests are filed, a hearing may be held. Appeals from the decision of the State Engineer may be taken to the courts. The State Engineer can establish ground-water districts, can set limits on the quantity of water appropriated in a district, and can exercise police power to prevent waste. "Beneficial use" being the basis of the right to use water, an appropriation is effective only as long as it is used beneficially, and water rights not exercised for a period of four consecutive years are forfeited.

The careful administration of this law has prevented the overexpansion of certain ground-water areas; and, in other areas where serious depletion of the water supply was threatened, rehabilitation has commenced.

RAPHAEL G. KAZMANN,[10] ASSOC. M. AM. SOC. C. E.[10a]—A distinct contribution to ground-water hydrology is presented in this paper because it discusses, from an engineering viewpoint, a subject that has been all too neglected in the past by the majority of civil engineers. However, the self-imposed restrictions on the scope of the paper are too severe—many of the concepts outlined are applicable east of the 97th meridian and are useful to industry as well as to agriculture. Limitation 2 (see "Introduction") deserves to be quoted again in full, as it expresses conditions which are also necessary for the successful use of ground water by industry.

> "2. Only these situations in which the following conditions exist are considered:
> "(*a*) Ground water in the alluvium is readily accessible to extraction by pumps;
> "(*b*) The alluvium is sufficiently permeable to yield water in commercial quantities;
> "(*c*) The alluvium is naturally charged with water or is susceptible of being charged artificially to such an extent that heavy commercial drafts can be sustained; and
> "(*d*) If artificial charging is necessary it can be done at feasible cost."

In the eastern part of the United States, many deposits of alluvium furnish large supplies of ground water; and, in the northern states, many river valleys are filled with glacial outwash that may be a productive source of water. Many of these valleys are largely or wholly untouched by industrial or agricultural ground-water developments, thus making the problem of the safe yield from a "virgin valley" a very real one.

With regard to the industrial development of ground water, the major limiting factors are hydrologic and not economic. The utility of ground water

[10] Res. Engr. (Hydr. Engr. P-2), U. S. Geological Survey, Memphis, Tenn.

[10a] Received by the Secretary May 29, 1945.

for industrial purposes is so great that the cost of lifting the necessary quantities an additional distance is generally insignificant as compared with the cost of replacing the ground-water supply. Furthermore, this water is either discharged in sewers or, when uncontaminated, is returned to water-bearing sand through recharge wells, as in Long Island, New York (57).[10b]

The methods used by Mr. Conkling in the determination of the safe yield from a valley are based on the sampling of material followed by laboratory work. As is recognized in the paper, this procedure is subject to large errors.

The most fundamental difficulty in applying the results of laboratory analyses to field conditions is the tacit assumption that the sum of the properties of the parts of a water-bearing formation is equal to the properties of the formation as a whole. This is seldom, if ever, true. The sum of the properties of the parts of any complex unit is usually something different from the properties of the entire unit. This can be illustrated by an analogy: An automobile is far more than the equivalent of the sum of the unassembled parts—it has properties gained from the relation of the parts acting together that the parts, unassembled, do not have. In much the same way an aquifer, as a unit, has properties different from the properties of the materials comprising it—when the individual properties are determined and added in any of a number of complicated ways. This basic difficulty remains no matter how carefully the samples are taken. There are other difficulties, such as the problem of obtaining representative samples and the disturbance of the sample as it is brought into the laboratory for analysis. This latter condition is especially true of unconsolidated materials.

A questionable procedure should not be justified merely by stating that it is followed in determining other physical properties. The procedure may be satisfactory for determining some physical properties and yet be unsatisfactory for determining hydrologic coefficients. To carry the previous analogy a step further: The weight of the car is equal to the sum of the weights of the parts. However, without testing the car in an assembled condition, how can one determine how far the car will go or what horsepower will be developed at 50 miles per hour?

There are methods for testing materials in place and determining hydrologic coefficients such as permeability and specific yield without relying on samples for quantitative determinations. Such methods can be applied profitably to alluvial deposits. For example, the problem of the safe yield of an essentially "virgin valley" was encountered in 1942 by the writer and associates in the valley of the Miami River south of Hamilton, Ohio (58). As the work was done under circumstances typifying the industrial utilization of ground water in the east, details may be of interest. The estimates of the safe yield were based on determinations of hydrologic coefficients of the aquifer in place, resulting in an increase in the reliability of the results as compared with those based on laboratory determinations.

The problem started in the Mill Creek Valley near Cincinnati, Ohio, where large industrial plants were in operation—partly because of the plentiful supply

[10b] Numerals in parentheses, thus: (57), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

of ground water. By 1942 the industries and the municipalities in suburban Cincinnati were pumping more than 16 mgd from the water-bearing alluvium in the valley. The cumulative effect of the pumping was to lower ground-water levels in some parts of the area as much as 90 ft over a period of 50 years. By 1942 only 40 ft of saturated, water-bearing sand remained in some of the most heavily pumped areas. In others a saturated thickness of 60 ft or 70 ft remained.

In 1941 a new plant for the manufacture of airplane engines was built in the Mill Creek Valley just north of Lockland, Ohio. Even as originally designed, the plant was a giant one. Then, under the forced draft of war, its capacity was doubled. The water requirements, already high when the available ground-water supply was considered, also climbed. Before the attack on Pearl Harbor, Hawaii, in 1941, the water source for the big engine plant was a battery of large diameter wells tapping the valley fill underlying the plant property.

As water demands rose, water levels in the vicinity of the plant declined rapidly; and, partly on the basis of previous work by the U. S. Geological Survey, it was decided to install a new well field in the outwash-filled valley of the Miami River south of Hamilton, Ohio, and to abandon the well field at the plant except for emergency service. This plan necessitated the construction of about 20 miles of 42-in. and 36-in. pipe line, which, together with smaller pipe lines, wells, pumps, and motors, cost nearly three million dollars. This sum indicates the industrial value of cool, clean, ground water.

The Federal Works Agency, which was to construct the well field and pipe line, requested the Geological Survey to determine the effects of the new well field on existing ground-water supplies in the near-by Hamilton area and to make estimates of the perennial safe yield of the well-field area. In this connection, "safe yield" was defined as in Mr. Conkling's definition (see heading, "Safe Yield: Definitions"):

> "* * * the annual extraction from a ground-water unit which will not, or does not—
>
> "1. Exceed the average annual recharge * * *."

During the course of the investigation, thirty-two new wells were drilled in the area of the well field, including twelve production wells, and the geologic information obtained was analyzed carefully to delineate the area and the thickness of the ground-water reservoir. A number of farm and industrial wells that had been located and measured during an earlier investigation yielded valuable water-level information, as well as some geologic data (59)(60).

Water levels in the Miami Valley south of Hamilton had been measured periodically since 1939 and hydrographs of a number of observation wells were available, as illustrated in Fig. 10. A study of these graphs revealed an average annual water-level fluctuation of 8 ft.

The ground-water reservoir is recharged each spring by water from rain and snow, following which the water levels slowly decline as the water is evaporated and transpired while moving toward the Miami River. In 1941 and 1942,

which were very dry years, the water levels did not recover normally, but in 1943 they rose to about the 1940 levels.

Maps of the piezometric surface were prepared and these showed conclusively that ground water south of Hamilton was feeding the Miami River.

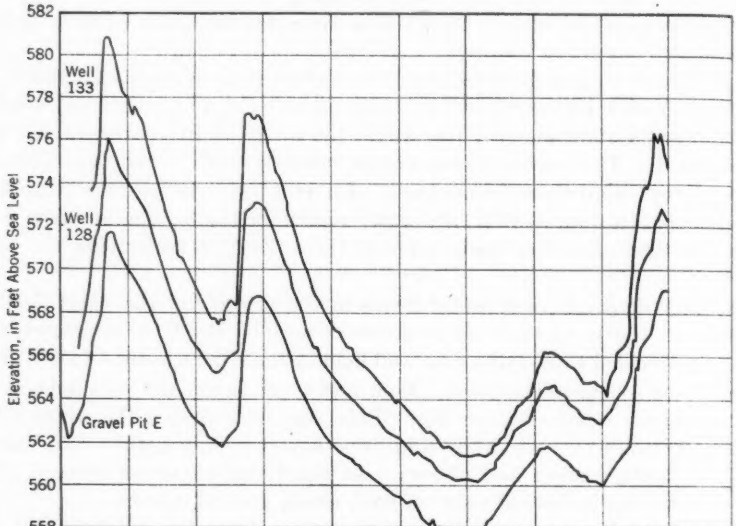


Fig. 10.—Typical Hydrographs of Wells and Gravel Pits in the Area South of Hamilton, Ohio

Fig. 11 is such a contour map. The slope of the water table is toward the Miami River. There was no heavy pumping in the area concerned, although large quantities of water have been pumped for many years in Hamilton, and this pumping undoubtedly has had some effect on the water levels in the area to the south. In the latter area, the inhabitants used the ground-water supply only for domestic purposes and for stock. At the time of the investigation, there was no development for irrigation.

To translate the available information into useful computations of safe yield, two pumping test investigations were conducted using methods developed by C. V. Theis (61) and described by L. K. Wenzel, Assoc. M. Am. Soc. C. E. (62). The method of investigation consists of sinking observation wells on a line from a well that is being pumped and observing the drawdowns that occur after pumping begins. From the discharge rate, the shape of the cone of depression at different times, and the rate of decline of the piezometric surface at the several observation wells, it is possible to compute the transmissibility of the water-bearing formation and the specific yield, or coefficient of storage, of the aquifer.

Using the quantities obtained from such pumping tests, two independent methods of computing safe yield were available:

1. By use of maps of the piezometric surface, together with the related geologic information, the average gradient and cross-sectional area were determined. The average gradient multipled by the cross-sectional area and the coefficient of transmissibility on any line perpendicular to the direction of ground-water flow gave the daily rate of flow through the cross section.

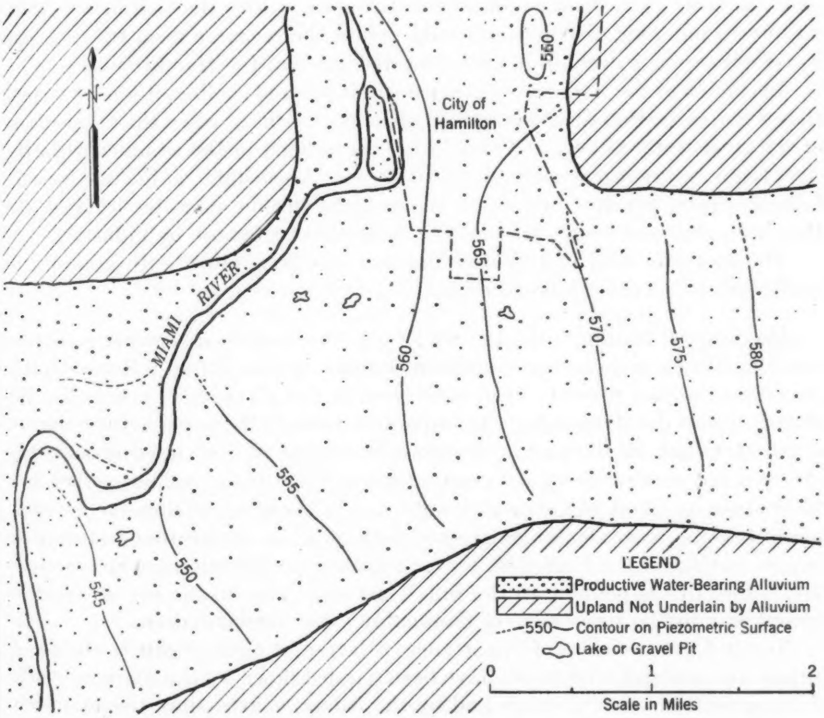


FIG. 11.—PIEZOMETRIC CONTOUR MAP OF AREA OF THE PROPOSED WELL FIELD SOUTH OF HAMILTON, OHIO, IN APRIL, 1943

2. The preceding computation should give results similar to those obtained by using the product of the specific yield times the average annual rise of the water level, multiplied by the up-gradient area between the given cross-section line and the ground-water divide, divided by 365 to convert the result to a daily basis. Such an assumption is based on the theory that precipitation is the source of the ground water and that, after reaching the water table, the water flows toward the Miami River, as indicated by contour maps.

The results were in reasonable agreement, and it was found that an average of about 13 mgd would be available from precipitation alone. However, the results of one of the pumping tests showed that, by lowering the ground-water

level near the Miami River, water would infiltrate from the river into the aquifer, thus increasing appreciably the indicated safe yield of the area. From the geometry of the new well field, computations were made which showed that a minimum of 6.5 mgd could be expected to infiltrate from the river over a long period of well-field operation. This value was undoubtedly conservative as it did not include the effects of occasional floods which cover part of the well-field area—undoubtedly helping to replenish the ground-water reservoir.

The quantity of water recharged from the Miami River was found to depend, in large part, on the pumping distribution within the well field. Thus, an unwise pumping distribution would reduce the recharge from the river and cause unnecessary declines in ground-water levels in the Hamilton area.

Inasmuch as the water was to be used for industrial cooling, it was important that river infiltration should be encouraged in the winter when the temperature of the river water is low, and should be discouraged in the summer when the river temperature reaches from 85° F to 90° F. A schedule was devised for well-field operation which would cause the smallest decline in water levels in the Hamilton area and would produce the coolest water for use by industry.

The methods used in this investigation can be applied, with appropriate modifications, to other alluvial deposits.

DONALD M. BAKER,[11] M. AM. SOC. C. E.[11a]—A method of water conservation long known and exercised has been clarified by the author. However, this procedure has just recently been considered in the planning of comprehensive stream system developments. An important point in the paper is the reference to the fact that, in the past, ground-water storage in such developments has occurred not as a result of systematic foresight and planning, but rather as a by-product of direct utilization of waters. As these comprehensive developments become more common, greater reliance upon underground storage in certain sections of the United States for complete conservation can be expected; but, before this is possible to the fullest extent, rights to the use of water so conserved must be susceptible of reasonably exact determination.

Nearly 1,000 decisions of importance with respect to the rights to the use of surface and underground water have been handed down by the supreme courts of the seventeen western states and by the federal courts, since the admission of California into the Union in 1850. All these states, except Montana, have centralized agencies for the administration of surface waters, and in some states, of underground waters. These decisions, plus constitutional and legislative enactments, as well as practices and policies developed by state administrative officials, have fairly well crystallized the legal principles that relate to rights to the use of both surface and underground waters, when such waters occur as the result of the natural phenomena of precipitation and runoff.

The doctrines of appropriation and of riparian rights for surface waters and underground waters in known and defined channels are well established. The former is recognized in some states, whereas both doctrines exist side by side in others. In the case of percolating underground water, the "common

[11] (Ruscardon Engrs.), Los Angeles, Calif.

[11a] Received by the Secretary May 29, 1945.

law" doctrine, under which the landowner is presumed to own all water under his land as part and parcel of the soil thereof, the doctrine of correlative rights, more or less analogous to that of riparian rights in surface flow, and the doctrine of appropriation—all have been reasonably clearly defined.

When water occurs in underground storage as a result of man's activities and when it is extracted for beneficial purposes or finds its way into a natural stream, augmenting the flow thereof before diversion, the rights to use of such waters are not so clearly established. Most cases that have reached the higher courts as a result of controversies over the foregoing types of waters have dealt with return flow from irrigated lands, which has reached either an artificial or a natural drainage channel. In some states, those responsible for the occurrence of such waters are allowed to dispose of them to others. In other states, once the waters reach a natural stream—particularly when they have passed beyond the limits of the area from whose irrigation they result—they are considered part and parcel of the stream. Nearly all the decisions relating to this type of controversy have arisen in those states where the riparian doctrine does not exist.

During the next decade or two, however—particularly when a period of deficient precipitation and stream flow occurs—situations resulting from developments initiated since 1925 may be expected to cause serious controversies. In some instances, difficulties may be obviated by legislative enactments or constitutional amendments, whereas in others they must be settled by court decisions.

One factor to which the courts in many states appear to give consideration is the intent of the agency that is responsible for the occurrence of water in underground storage. If the agency has ignored the existence of such water—usually return flow, although sometimes surface waste—as an augmented supply to an underground basin or to a surface stream, the tendency has been to allow others to assert and secure rights to the supply. On the other hand, if the agency responsible for the occurrence of the water has consistently claimed the right to the use of the water, it has usually been allowed. Another principle, which has been favored with but little recognition by the courts, is that of the "foreignness in time" of return waters which augment natural stream flow.

Waters, originating outside a drainage basin and entering a stream, are considered as "foreign." In some states, riparian rights are not attached to the use of these "foreign" waters, since such waters are not a part of the natural flow of the stream. Somewhat similarly, an appropriator, when he made his appropriation and planned his development and use of water, might consider as available to him such waters of the natural flow of the stream as existed over and above existing prior rights. If the natural flow were augmented by artificial means, it would seem that such water should not inure to the appropriator, but should be subject to appropriation by the first comer. The return water, appearing and augmenting the natural flow, although water of the stream in question, nevertheless is no more a part of the natural flow occurring at the time than water from a distant drainage basin is part of the natural flow of the stream. However, most of the higher courts where the subject has been considered do not agree with this opinion.

The following problems may be considered as representative of many that may be expected during the next decade or so:

1. A flood control district gradually releases flood waters from its reservoirs, a considerable portion of these seeping from the stream channel into an underground basin, thus resulting in a substantial increase in the normal replenishment of such a basin. Prior to the construction of the flood control reservoirs, replenishment of the basin had been in excess of draft; and, after adjudication, the pumping rights had been substantially reduced. In the adjudication proceeding, a substantial area of land within the basin was enjoined from ever using ground water. Does this augmented supply benefit those pumpers whose pumping rights were reduced in the adjudication? Does it inure to those others who were enjoined because of an inadequate supply? In other words, how should the supply be apportioned?

2. In a situation similar to that in problem 1, the increased replenishment caused a substantial increase in surface flow in a lower constricted section of the stream during the period of low flow. Does this augmented supply in the stream benefit existing appropriators in this section, or riparians, or may any one secure an appropriative right by filing a claim?

3. In a given underground basin, pumping draft over a period of years equals replenishment, and pumping rights have been adjudicated and restricted upon this basis. Gravity water is brought into the basin from a distant source, and return water from this source materially increases the replenishment. Can the original pumpers from the basin increase their draft because of the augmented supply, or are they limited to a draft equal to previous natural replenishment? In the latter case, could owners of overlying land, previously precluded from pumping because of deficiency in supply, utilize the return water, or would the agency importing the water have the first right to its recovery and use?

4. An irrigation district stores winter and early spring flood waters and uses them for irrigation. At the time plans were being prepared for the project, it was expected that the summer flow of the stream, which is inadequate to supply existing lower appropriative rights, would be augmented by return flow. Therefore, the district filed a claim on such augmented flow at the time of the original storage filing, for use on lands in the lower reaches of the project. Can the district secure a prior appropriative right to this augmented flow, or does such flow inure to existing and prior lower appropriators or to riparians to augment deficient summer supply?

5. Water from a surface stream is "spread," or placed in underground storage artificially, by an agency that expects to recover it later from wells. Because of physical conditions, not all the water so spread can be recovered by the agency doing the spreading. A party who has been exporting water from the basin is physically able to secure the remainder not recovered by the agency spreading it. However, certain overlying pumpers also claim this augmented supply. Who has the right to such water?

These and similar problems will be before the courts in the future. Sound legislation, passed to settle such controversies before they arise, or intelligent decisions, based upon an informed knowledge of the technical and economical

features involved in the cases as well as upon legal precedent, will do much to crystallize the legal principles applying to water occurring in underground storage as a result of man's activities, and will encourage conservation. Much knowledge has become available concerning the occurrence and behavior of ground water since 1925 or 1930, and the fact of the existence of ground water should allow legislatures and courts to act with far more assurance than they were able to do shortly after the turn of the century.

HYDE FORBES,[12] M. AM. SOC. C. E.[12a]—This discussion of ground-water utilization provides an excellent over-all view of the objectives and accomplishments of such work in the west. The geologically modern alluvial fans built up by the major streams of the California Central Valley, southwestern Arizona, western Nevada, parts of Oregon, and south-central Washington and the alluvium filling of geological structural basins of mountain valleys and coastal plains of the west provide underground storage reservoirs whose utilization in connection with the development of the surface water resources should be a requirement in drainage basin development. Such reservoirs aid in the irrigation of less favorably situated lands and, in some instances, in the control of contamination of soils through alkali concentrations caused by the excessive accumulation of ground water.

In the "Summary," the author states:

> "This paper has presented, briefly, some of the values found and methods so far achieved, keeping in mind always that a solution of any water problem of the nature discussed in this paper [ground water] is accomplished by merely substituting correct values in a simple equation. It is the values which are often difficult to determine correctly; but as data accumulate this difficulty decreases."

Such a statement is the essence of all investigation involving underground conditions; and the writer finds, as the data accumulate, that preconceived simple relationships are not in reality simple and that the complexity of factors and of values increases. However, simple relationships or simple equations are a good starting point, subject to modification as more is learned about each specific problem.

In connection with the California State Wide Plan (later known as the Central Valley Project), in 1930 the writer undertook an investigation of the geology and underground water storage capacity of the Sacramento (63) and San Joaquin valleys (22*a*). Drainage factors for the ground-water reservoirs were assigned on the basis of grain size of the alluvial materials comprising broad areas as determined from penetration records of wells bored in those areas. The total capacity of underground storage was based on the extent of materials of a water-absorptive character delimited through an areal geological survey and on the practical limitations of water-table fluctuations. The recharge values were assumed as average seepage contributions, determined by test and measurement.

---

[12] Cons. Engr. and Geologist, San Francisco, Calif.

[12a] Received by the Secretary June 9, 1945.

The writer's conception of the problem was that ground-water reservoirs (inflow and yield) could be treated like surface reservoirs—once the factors relative to capacity and yield had been assigned. However, the years since 1930 have produced data which indicate that the storage capacity, as measured by yield of most basins, is greatly reduced with depth below ground surface due to compaction under overburden load. The storage capacity is further reduced by the compaction of the material taking place after the first dewatering, and it is to be expected that further compaction will occur with each substantial lowering of water levels after recharge.

The increase in density of the water-bearing materials and decrease in yield as the depth below ground surface increases were determined quantitatively during the drilling of the plant 6 well for the City of Lodi in the Mokelumne River area of the California Central Valley. A continuous core was obtained and examined for compaction density and cementation in connection with pump tests and water level measurements made in 1937. When the well was 92 ft deep and had a water column of 66.6 ft, it produced an average of 65.6 gal per min of water per foot of drawdown, at discharge rates varying from 618 gal per min to 2,280 gal per min. The zone of pumping influence extended over a maximum area of 0.315 sq miles. The yield factor per foot of water column drawn upon $\left(\frac{66.6}{65.6}\right)$ was close to 1.00.

The well was driven to 270 ft and the pumping tests were repeated, extending the area of the zone of pumping influence to 0.546 sq miles and reducing the drawdown, but giving a resultant yield factor of 0.40. The final depth was 450 ft with a maximum pumping influence of 0.641 sq miles and a yield factor of 0.30—or less than one third of the yield in gallons per minute per foot of drawdown per foot of well column obtained at shallow depth. A current meter in an orifice was lowered into the well during the pumping tests at the 270-ft and 450-ft depths to measure the water yield of each section of the coarse sand developed. The results showed that the yield per foot of sand diminished rapidly with depth. More than half of the total well yield came from the top 66.6 ft of the column which contained eleven lenses of sand totaling 29 ft in thickness. The well bore hole between 92 ft and 270 ft intercepted twenty-six lenses of sand, totaling 46 ft in thickness, and the bottom 180 ft of the bore hole contained a total thickness of 58 ft of coarse sand lenses.

It is probable that yields from underground reservoirs will diminish with each foot of water-table lowering. The degree of diminishing return will depend on the character of the material comprising the reservoir. Each ground-water basin is a problem in itself and it is doubtful that values obtained from one area or combination of geological conditions are applicable to other areas. The question is: How much compaction, if any, will take place in the upper portion of the column where overburden load is not great and where seasonal fluctuation of the water table due to recharge and draft is effective?

The example of the subsidence of the Santa Clara Valley around San Francisco Bay in California due to ground-water drainage has been cited

frequently—namely, 1 in. at Redwood City, 0.5 ft at Palo Alto, and 4.1 ft at San Jose in the 12 years from 1920 to 1932. Further subsidence was measured in the 6 years from 1932 to 1938 during which time conservation dams were constructed and spreading recharge was practiced (for the last 3 years) over the upper stream fans. This subsidence was measured as 0.15 ft at Palo Alto and 1.6 ft at San Jose.

The water-table levels of well-field area for the City of Palo Alto had declined to a critical elevation; and, by 1937, the water drawn upon was of poor quality. Water was purchased from the municipal supply of San Francisco and pump draft was reduced more than half. The wet years from 1940 through 1942 rapidly raised the well-field water levels. The writer has obtained the seasonal fluctuation of water levels in numerous wells in and surrounding the city well field since 1933. The plotted hydrologic pattern of these wells has been closely repetitive year after year, declining from 1933 through 1937 and rising in point of elevation from 1938 through 1942 until the fall of 1943 when the pattern broke and a decline started which has continued through the dry years of 1944 and 1945.

Pumping was further reduced in 1944 but the decline continued. It is not possible to obtain an exact quantitative measure; but, during the 5 years preceding October, 1937, the Palo Alto pump draft averaged 819.7 million gal per season and the average seasonal replenishment was 70.8% of normal—the water level of the ground-water reservoir was lowered 4 ft, or only 0.8 ft per season (about 0.1 ft per 100 million gal pumped). During the following 5 years, when the supplemental supply was brought in, the average seasonal pumpage amounted to 425 million gal per season and the average seasonal replenishment was 110% of normal. The basin water level rose 48.5 ft, or an average of 9.7 ft per season. During the season of 1943–1944 replenishment was 68.5% of normal, comparable to the first 5-yr period of record, and pump draft was reduced to 388 million gal. The basin water level lowered an average of 5.5 ft or 1.4 ft per million gal pumped as against 0.1 ft per million gal of pump draft when the basin was first dewatered. These data indicate that there is a substantial loss of storage capacity due to compaction upon drainage from ground-water storage. This capacity may not be recovered when the water levels rise.

A considerable area in California has been devoted to ground-water recharge through spreading, particularly since 1930, and a large amount of data has been accumulated relative to specific test plots, methods, and effects. The author has mentioned some of the more general results and difficulties that have been encountered. Recharge is readily effected in the ground-water areas most intimately connected with and adjacent to large surface streams. The flow of such streams, possibly needing regulation, is adequate to serve all the lands and purposes. On the other hand, ground-water basins provided by the limited alluvial cones of small intermittent streams, or interstream areas are those wherein ground-water resources are of the most value, have been most heavily drawn upon, and, in many localities, are critically exhausted. The problem of recharge of such areas is far from solution at present.

The ramifications of the general problem of ground-water utilization, draft, and recharge, cover many scientific fields and economic and legal considerations. In stream system development, an exchange of water—service from surface water over areas of ground-water depletion and to a considerable extent from wells for areas bordering major streams of the west—should be the answer. The economic adjustment and legal difficulties, when property or water rights are involved, are probably insurmountable.

Bibliography.—

(22) **"San Joaquin River Basin,"** by A. D. Edmonston, *Bulletin No. 29*, Div. of Water Resources, Dept. of Public Works, State of California, Sacramento, Calif., 1931. (*a*) Appendix B.

(57) **"Ground-Water Temperature on Long Island, New York, as Affected by Recharge of Warm Water,"** by M. L. Brashears, *Economic Geology*, Vol. 36, 1941, pp. 811–828.

(58) **"A Quantitative Study of the Well Field of the Mill Creek Valley Water Supply Project, Butler County, Ohio,"** by Fred H. Klaer, Jr., and Raphael G. Kazmann, July, 1943 (unpublished).

(59) **"Progress Report on Ground Water Investigation in Butler and Hamilton Counties, Ohio,"** by D. G. Thompson and F. H. Klaer, Jr., U. S. Geological Survey, February 19, 1940 (unpublished).

(60) **"Ground Water Resources of the Cincinnati Area in Butler and Hamilton Counties, Ohio,"** by F. H. Klaer, Jr., and D. G. Thompson, U. S. Geological Survey, June 26, 1941 (publication pending).

(61) **"The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage,"** by C. V. Theis, *Transactions*, Am. Geophysical Union, 1935, pp. 519–524.

(62) **"Methods for Determining Permeability of Water-Bearing Materials,"** by L. K. Wenzel, *Water-Supply Paper No. 887*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1942.

(63) **"Sacramento River Basin,"** *Bulletin No. 26*, Div. of Water Resources, Dept. of Public Works, State of California, Sacramento, Calif., 1931, Appendix F.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE MECHANISM OF ENERGY LOSS IN FLUID FRICTION

Discussion

BY GEORGE R. RICH, J. C. STEVENS, AND LEON BESKIN

GEORGE R. RICH,[32] M. AM. SOC. C. E.[32a]—In strikingly clear and compact form, the paper presents the spatial implications of the differential energy balance for representative problems in which the flow is directed by a wall or plate. This same differential expression also appears to afford a ready method of calculating similar energy exchanges for cases without confining walls, such as wakes behind obstacles and jets discharging into still fluid.

The requisite velocity distribution for the jet discharging into still fluid under the condition of laminar flow is given in Fig. 27 and the formula:[33]

$$u = 0.4543 \left(\frac{M^2}{\rho^2\, \nu\, x} \right)^{1/3} \operatorname{sech}^2 \left[0.2752 \left(\frac{M}{\rho\, \nu^2} \right)^{1/3} \frac{y}{x^{2/3}} \right] \ldots\ldots (92)$$

in which M, the total momentum flowing across a section, is the same for all sections x and is equal to

$$M = 2\,\rho \int_0^\infty u^2\, dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

Since the present investigation is concerned only with changes in the variable y, Eq. 92 may be written:

$$u = C_1 \operatorname{sech}^2 (C_2\, y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

By direct substitution in Eqs. 11 and 12a and integrating with respect to y:

$$\left| W_s \right|_0^y = 4\,\mu\, C^2_1\, C_2 \int_0^y \operatorname{sech}^4 (C_2\, y) \tanh^2 (C_2\, y)\, C_2\, dy$$

$$= 2\,\mu\, C^2_1\, C_2 \left[\frac{64}{5\,(e^{2C_2 y}+1)^5} - \frac{32}{(e^{2C_2 y}+1)^4} + \frac{80}{3\,(e^{2C_2 y}+1)^3} - \frac{8}{(e^{2C_2 y}+1)^2} \right]_0^y \ldots (95a)$$

NOTE.—This paper by Boris A. Bakhmeteff and William Allan was published in February, 1945, *Proceedings*.

[32] Chf. Design Engr., TVA, Knoxville, Tenn.

[32a] Received by the Secretary May 21, 1945.

[33] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. I, p. 145.

and

$$\left| W_B \right|_0^y = 2\,\mu\, C_1^2\, C_2 \int_0^y \left[\operatorname{sech}^6 (C_2\, y) - \operatorname{sech}^4 (C_2\, y) \tanh^2 (C_2\, y)\right] C_2\, dy$$

$$= 2\,\mu\, C_1^2\, C_2 \left[-\frac{96}{5\,(e^{2C_2 y}+1)^5} + \frac{48}{(e^{2C_2 y}+1)^4} - \frac{112}{3\,(e^{2C_2 y}+1)^3} + \frac{8}{(e^{2C_2 y}+1)^2} \right]_0^y \ldots (95b)$$

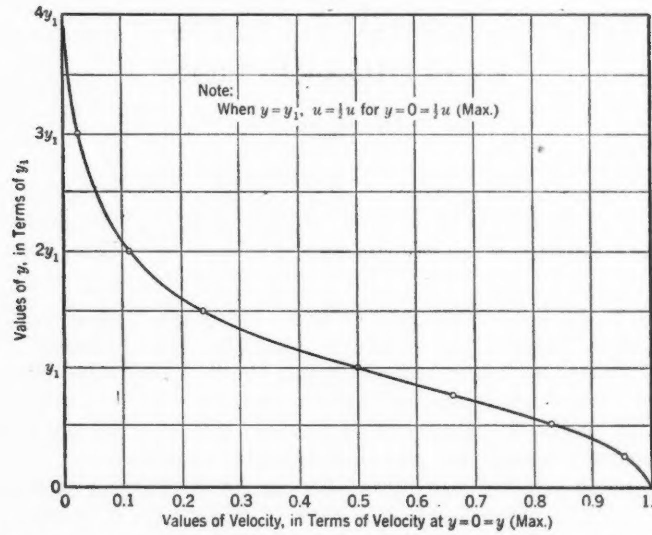


FIG. 27.—VELOCITY DISTRIBUTION FOR RECTANGULAR JET DISCHARGING INTO STILL WATER; LAMINAR FLOW

The resultant energy exchanges are given in Fig. 28(*a*), in which the characteristic shape of the borrowing curve W_B reflects the abstraction of kinetic energy from the central zone of the jet—not only for the subsequent work of viscous deformation at the particular section x, but also for imparting velocity to water at increasingly higher ordinates as the jet diverges progressively in the x-direction. This salient feature may be inferred from the curvature of the velocity distribution graph, Fig. 27; thus:

$$K = \frac{\dfrac{d^2y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} \ldots (96)$$

In the vicinity of the jet axis $y = 0$, the curvature is convex upward, changing to convex downward at a point of inflection somewhere about $y = 0.75\, y_1$. Thus, $\dfrac{d^2y}{dx^2}\left(\text{or } \dfrac{d\tau}{dy}, \text{ which measures the rate of borrowing}\right)$ must change algebraic sign near $y = 0.75\, y_1$; and, consequently, the summation of borrowed energy W_B must decrease, finally approaching, asymptotically with W_s, the value of 0.745.

The corresponding physical picture appears to be that, at low values of y, the shearing force on any element from the moving water below exceeds the opposing shear due to the inertia of the relatively slower moving water above, and the sign of the resultant increment of borrowed energy is positive. At

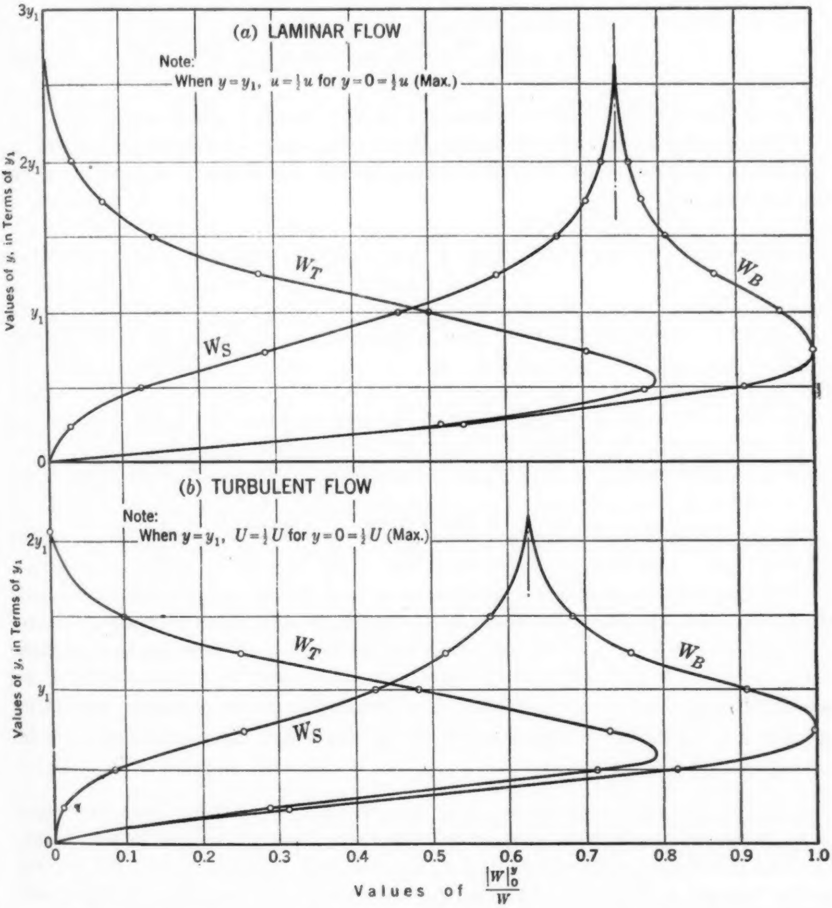


Fig. 28.—Energy Exchanges for Rectangular Jet Discharging into Still Water

higher values, the inertia shear from above exceeds the kinetic shear from below the element, and the algebraic sign of the increment becomes negative.

For the case of the rectangular jet discharging into still fluid under conditions of turbulent flow, the actual equation for the longitudinal velocity, although it involves only elementary functions, is exceedingly cumbersome.[34,35]

[34] "Berechnung Turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 6, 1926, pp. 468–478.

[35] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. II, p. 594.

Since the actual distribution of velocity is very much the same as that for the laminar condition, Fig. 27 and Eq. 92 may be assumed sufficiently accurate for present purposes in calculating energy exchanges under the turbulent condition.

Under turbulent flow let τ be the virtual viscous, or Reynolds' shearing, stress,[36] expressed in accordance with the momentum transfer theory as

$$\tau = \rho\, l^2 \left(\frac{\partial U}{\partial y}\right)^2 = \rho\, c^2\, x^2 \left(\frac{\partial U}{\partial y}\right)^2 \quad\ldots\ldots\ldots\ldots(97)$$

in which the mixing length l is assumed to be $l = c\, x$, c being constant.

The symbol U denotes the longitudinal component of the mean velocity at ordinate y and abscissa x. Substituting in the appropriate equations of the energy balance:

$$\left| W_s \right|_0^y = 8\, C^3{}_1\, C^2{}_2\, c^2\, x^2\, \rho \int_0^y \operatorname{sech}^6 (C_2\, y) \tanh^3 (C_2\, y)\, C_2\, dy$$

$$= 8\, C^3{}_1\, C^2{}_2\, c^2\, x^2\, \rho \left[\frac{\operatorname{sech}^8 (C_2\, y)}{8} - \frac{\operatorname{sech}^6 (C_2\, y)}{6} \right]_0^y \quad\ldots\ldots(98a)$$

and

$$\left| \mathrm{W}_B \right|_0^y = 8\, C^3{}_1\, C^2{}_2\, c^2\, x^2\, \rho \int_0^y$$

$$\times \left[\operatorname{sech}^8 (C_2\, y) \tanh (C_2\, y) - 2 \operatorname{sech}^6 (C_2\, y) \tanh^3 (C_2\, y)\right] C_2\, dy$$

$$= 8\, C^3{}_1\, C^2{}_2\, c^2\, x^2\, \rho \left[\frac{\operatorname{sech}^6 (C_2\, y)}{3} - \frac{3 \operatorname{sech}^8 (C_2\, y)}{8} \right]_0^y \quad\ldots\ldots(98b)$$

Fig. 28(*b*) shows the corresponding energy exchanges. In comparison with the laminar jet, the chief difference is that a relatively greater portion of kinetic energy is convected from the zone of the central axis $y = 0$ for use in imparting velocity at the higher ordinates in sections subsequent to x and also that the spending at section x is completed at a somewhat lower ordinate, indicating greater concentration of spending in the mixing zone, as would naturally be anticipated.

J. C. Stevens,[37] President, Am. Soc. C. E.[37a]—Entirely novel concepts of the internal mechanism of fluid friction have been presented in this paper. In 1942 the senior author accepted an assignment, heading a research project on the Nature of Hydraulic Friction for the Society's Committee on Hydraulic Research, of which the writer was then chairman. The research committee felt that it was high time to leave the beaten path of empiricism and to step boldly into newer fields of inquiry in fundamental concepts if further advancement in fluid mechanics was to be realized. This paper, although largely a mathematical analysis of viscous forces and their resultant effects, will serve as a beacon to direct the trend of future experimental studies in laboratory and field. From this analysis and from the physical research that it will

[36] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. II, Eqs. 52 and 53, p. 582; and Eq. 97, p. 592.

[37] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[37a] Received by the Secretary May 31, 1945.

stimulate, new methods of evaluating flow characteristics of liquids and gases are certain to be evolved.

The suggestion that the principal difference between laminar and turbulent flow may be one of vorticity scale seems to conform with observed phenomena. In pure laminar flow, the vorticity induced by viscous shear is microscopic, even molecular, for which the conversion of energy into heat is definitely confined to the shear plane. As velocities increase, flow characteristics gradually undergo a transition into a state of turbulence. One might indicate the transitional stages by such words as "pure laminar," "laminar," "mildly turbulent," "turbulent," and "shooting." Each progressive stage is characterized by a corresponding increase in the size of the vortices formed by increasing stresses induced by increasing viscous attrition following each increment in the over-all velocity. At least, this is a comforting explanation of why the losses seem to be proportional to an increasing power of the velocity exponent, from 1 for pure laminar to 2 for shooting flow.

It is well to recognize that the analysis presented applies only to steady uniform flow, whether laminar or turbulent. The case of steady nonuniform flow offers a challenge that can be met only by experimentation. A. A. Kalinske,[38] Assoc. M. Am. Soc. C. E., has encountered this problem in the case of dilating tubes in which the flow is constant but in which velocities are being reduced, thus converting kinetic energy into potential energy. It is well known that, through reducers where velocities are increasing, losses from turbulence are quite low. In fact, such losses may be less than those for steady uniform flow through a corresponding length of an equivalent tube of constant diameter. Therein lies a neat little research problem. If the direction of flow is reversed, however, the losses become inordinately high. This is due to excessive turbulence in the vortex generating zone which, owing to receding boundaries, now occupies a much greater portion of the total cross-sectional area than is the case for uniform flow in a conduit of constant area.

In the example of the large river, the vertical velocity curve, Fig. 20(*b*), does not show the customary lessening of velocity at the surface. If the curve did show this typical characteristic, the cumulative curves of Fig. 20(*c*) and Fig. 21 would be slightly different. If the vertical velocity curve has a lesser surface velocity than occurs at a point below the surface, the effect is to make the "borrow" curve less than the "transfer" curve. This cannot be true for it would mean that more energy is transferred than is available.

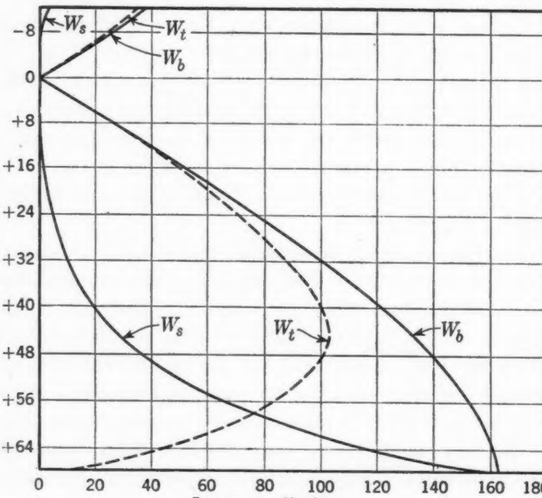
Some vertical velocity curves do not show a lessening of velocity at the surface, but many curves do evidence such a lessening.[39] The writer was curious to know what effect this would have on the "borrow," "transfer," and "spending" curves. He therefore selected one of many vertical velocity curves taken by the U. S. Geological Survey on Columbia River at The Dalles, Oregon. The curve is shown in Fig. 29. The banks of the river are steep basalt cliffs with submerged talus. The stream bed is very rough and is strewed with large boulders. Depths are great and velocities low. The

[38] "Conversion of Kinetic to Potential Energy in Flow Expansions," by A. A. Kalinske, *Proceedings*, Am. Soc. C. E., December, 1944, p. 1545.

[39] "River Discharge," by J. C. Hoyt and N. C. Grover, John Wiley & Sons Inc., New York, N. Y., 1920, p. 52.

vertical velocity curve is quite typical of such curves taken at this site near midstream.

To avoid the anomaly of transferring more energy than is available, the expedient of fixing the neutral axis at the depth of maximum velocity was adopted. This is exactly the procedure for flow in a pipe. The resulting curves of "borrow," "transfer," and "spending" are shown in Fig. 30. The



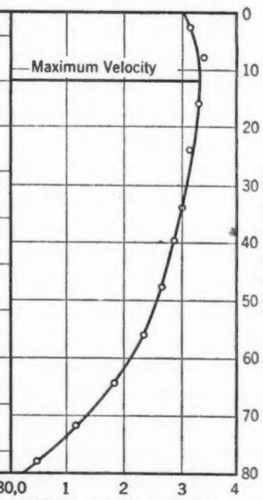


FIG. 29.—VERTICAL VELOCITY CURVE, STATION 395, AT THE DALLES, OREGON

FIG. 30.—ADJUSTMENT OF NEUTRAL AXIS TO AVOID THE ANOMALY OF TRANSFERRING MORE ENERGY THAN IS AVAILABLE

afore-mentioned anomaly is not in evidence, which seems to indicate that instead of the surface of an open stream being considered as the neutral axis for energy transfer, it would be better to use the thread of maximum velocity.

The writer hopes that this paper is only the beginning of a searching program of study and experimentation through which the fundamental nature of fluid resistance to flow will become better known and that practicable usage of the knowledge will speedily follow.

LEON BESKIN,[40] ASSOC. M. AM. SOC. C. E.[40a]—A thorough description of the well-known phenomena involved in fluid friction is contained in this paper. In the writer's opinion, however, the authors have failed to propose new or more simple physical or mathematical concepts of viscous flow, in a form that would be useful to the practicing engineer, who thus could rely less on "detailed experimental data" (see item 4 of the "Summary"), and more on theory.

The authors have not translated their analysis into general mathematical expressions, since their analytical expressions are given only in the case of a two-dimensional rectilinear permanent flow and (in a somewhat less complete

[40] Senior Stress Analyst, Eng. Dept., Consolidated Vultee Aircraft Corp., San Diego, Calif.

[40a] Received by the Secretary June 27, 1945.

form) in the case of a cylindrical rectilinear permanent flow—both with constant kinetic energy. For example, Eq. 91 which is offered as a complete picture of the "local energy balance" (see item 3 of "Summary"), actually applies only to two-dimensional flow. This "balance" equation is the well-known expression for the differentiation of a product:

$$d(\tau\, u) = \tau\, du + u\, d\tau \qquad (99)$$

If a cylindrical flow were considered, Eq. 91 would be:

$$\underset{(w_b)}{-\frac{u}{r}\frac{d(\tau\, r)}{dr}} = \underset{(w_s)}{\tau\frac{du}{dr}} + \underset{(w_t)}{\frac{d}{r\, dr}(-\, r\, \tau\, u)} \qquad (100)$$

Thus, the mathematical expressions of the new concepts are seen to be inadequate because their validity is limited to an extremely simple case of viscous flow. It is impossible to ascertain what the consequences would be if the authors had generalized their concepts for an arbitrary three-dimensional motion, and whether new information would thus be revealed as to the phenomenon of viscous motion. To the writer it seems likely that nothing much would be gained by generalizing these concepts because, although Stokes' dissipation function has an absolute nature, the authors' borrowing and transfer functions have only a relative nature. This fact can be demonstrated by a simple inspection of Eq. 91. Velocity u is measured in reference to some axes of coordinates. If the water is flowing in a conduit which is also moving, u can be defined either in reference to the conduit, or in reference to absolute axes. This does not change the expression of w_s, but modifies both w_b and w_t. Academic as it may seem, this fact has far-reaching consequences. For instance, assuming that the same flow pattern occurs in a stationary conduit, in a turbine, or in a centrifugal pump, both terms, w_b and w_t, have different values in each of these cases; but their difference is zero when viscosity is neglected (they are both equal to zero in this case) and has a definite value when viscosity is considered. It can also be shown that the reversal point has a purely relative nature. It moves toward the center of the conduit in the case of a pump, and reaches the boundaries in the case of a turbine. Instead of "reversal point" (see Section 11) it would be more correct to use consistently the term "reversal surface" (see Fig. 14), or at least the term "reversal curve" in a section.

To summarize: The concepts introduced in this paper are defined in terms that are too narrow and should be generalized. The definitions have no absolute significance and change with every change in the system of coordinate axes. To justify the practical usefulness of the ideas presented in this paper, the authors need to offer a comprehensive series of illustrative examples.

Corrections for *Transactions:* In February, 1945, *Proceedings*, on page 130, Fig. 1(*b*), and on page 131, Fig. 2(*b*), change all *W*'s (W_0, W_s, W_t, and W_b) to *w*'s; on page 150, change the abscissa caption of Fig. 17(*a*) to read "Values of $W_0 \frac{u_f}{U}$"; and on page 158, line 27, change "across AB equal to the energy" to "across AB and the energy."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION AND THE DESIGN OF SETTLING TANKS

Discussion

BY NORVAL E. ANDERSON, AND R. A. MULHOLLAND

NORVAL E. ANDERSON,[41] M. AM. SOC. C. E.[41a]—The mathematical analyses presented by Mr. Camp should prove useful in further studies of settling tanks. Such theoretical analyses are informative. However, the "Conclusions" appear abrupt, since they imply recommendation of types of tanks for primary settling and for final settling of activated sludge, without reference to the available operating and experimental data on plant-size units and with little relation to the theory in the case of final tanks.

Multiple-tray settling tanks, similar in principle to the tank proposed by the author for primary settling, have been patented and promoted for many years. The author might have shown why these tanks have had such limited acceptance and in what way his proposed tank may be more acceptable.

Most of Mr. Camp's conclusions on final settling tanks for the activated sludge process may be questioned on the basis of an extensive study of plant-size experiments and operating data, some results of which were presented[42] by the writer in October, 1944.

It is doubtful if the statement in Section 10 that "Fig. 18 illustrates current trends in the design of final settling tanks[39,40] * * *" is based on a general view of the field.

The author's statements to the effect that density currents would be less with shallow tanks are not confirmed by actual velocity measurements. Experiments[42] on 126-ft diameter tanks at the Chicago (Ill.) Southwest Plant (allowing the sludge blanket to build up to various depths, thereby giving the effect of different tank depths) showed that the velocity of the density current

NOTE.—This paper by Thomas R. Camp was published in April, 1945, *Proceedings*.

[41] Engr. of Treatment Plant Design, The San. Dist. of Chicago, Chicago, Ill.

[41a] Received by the Secretary June 7, 1945.

[42] "Design of Final Settling Tanks for Activated Sludge," by Norval E. Anderson, *Sewage Works Journal*, January, 1945, p. 50.

[39] "Operating Experiences in New York City," by Richard H. Gould, *ibid.*, Vol. 14, No. 1, 1942, p. 77.

[40] "Developments and Trends in Sewerage Practices of 1941," by Samuel A. Greeley and Paul Hansen, *Water Works and Sewerage*, Vol. 89, No. 2, 1942, p. 53.

was in an approximately inverse ratio to the flow depth. Also, relatively higher velocities of density current were found[42] in the rectangular final settling tanks at Columbus, Ohio, and at Wards Island, New York, where the ratio of length to depth is relatively large.

The author's argument for withdrawal of sludge at the effluent end of the tank is open to some question. He states (Section 10) that:

> "The usual practice of withdrawing sludge from the bottom at the inlet end results in a mixture of fresh sludge short-circuited from the density current and stale sludge scraped from the effluent end. Short-circuiting of the sludge may be avoided if it is withdrawn from the outlet end * * *."

Why should one avoid this so-called short-circuiting of fresh sludge when one of the most desirable features of a final settling tank is the withdrawal of fresh return sludge? The writer agrees[42] that activated sludge settles near the inlet of the tank. Thus, with a long rectangular tank and flight-conveyer-type sludge removal mechanisms as indicated in Fig. 18, the length of time that the dense sludge remains in the tank would be governed largely by the speed of the mechanism, resulting in sludge of greater average age than would be the case if the drawoff were near the inlet. With the drawoff near the inlet, the dense sludge is required to move a much shorter distance and only the lighter particles that settle at the effluent end need to move the full length of the tank. Also, it is possible that greater flocculation is obtained by withdrawing sludge near the inlet, since this causes the sludge to flow against the density current and affords more opportunities for contacts between particles, as stated by the author under "7. Flocculent Suspensions."

Elimination of connecting conduits between aeration tanks and final settling tanks "in order to avoid high inlet velocities and the destruction of floc" may be practical in a small plant. However, in a large plant such an arrangement results in an uneconomical layout with sacrifice of flexibility in operation, as shown by layouts and studies of similar arrangements made by The Sanitary District of Chicago in 1933 for plants with 136-mgd capacity and more. Furthermore, it appears that "destruction of floc by excessive velocity gradients" (last paragraph of Section 10) is not an important factor with aeration tank effluent. Comparative tests made at three activated sludge treatment works of The Sanitary District of Chicago showed practically no change in the "settleability" of aeration tank effluent—even after a free fall over a weir and passage through both a venturi meter and a considerable length of conduit and aerated channel.

R. A. MULHOLLAND,[43] ASSOC. M. AM. SOC. C. E.[43a]—Although most consulting engineers, from time to time, are faced with the problems of designing sedimentation tanks, it seems that in most cases these consultants do not delve completely into the theories involving sedimentation.

As a result of this method of design there is the continual problem of inadequate settling. This is particularly true of sewage treatment plants.

---

[43] Consultant, Austin Eng. Co., Austin, Tex.

[43a] Received by the Secretary July 23, 1945.

In most industrial processes using sedimentation, such as those involving chemical reaction, the designers seem to have a much better understanding of sedimentation. This can be seen by the results obtained in such plants. Concerning the design of grit chambers, emphasis should be placed on the sixth and seventh paragraphs of Section 2. With the exception of the very largest class of sewage treatment plants, one of the weakest links in the entire process is that of the grit chamber. This is particularly true of sewage treatment plants serving small towns and villages. Here the designers have been faced with controlling financial outlay to eliminate every cost possible. In many instances an excuse for a grit chamber has been installed which could not possibly function to collect any grit whatsoever, and has only been an added expense to the construction of the plant itself.

Mr. Camp states that geometric models for plant-scale tanks cannot be adapted to produce those effects, but are quite useful in the measurement of hydraulic short-circuiting; also, that settling analysis may be corrected for short-circuiting to predict performance in plant-scale settling tanks. It has been the writer's experience with model tanks that their flow characteristics can be correlated to full-scale flow characteristics and zones of sedimentation can be reproduced. It remains a matter of judgment and experience upon the part of the designer to adapt model results to full-scale operation. In regard to Mr. Camp's statement concerning the building of an adjoining tank without connecting conduits, the writer has knowledge of at least a dozen plants built along this pattern in recent years. Virtually all of the recently adapted Hay's process plants have been so arranged in current years. Such construction results in a decided economy, since dividing walls are merely web members of a rigid structure of sufficient strength to withstand the hydraulic load.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

Discussion

By A. A. Eremin

A. A. Eremin,[32] Assoc. M. Am. Soc. C. E.[32a]—By considering the elastic properties of soil the author has developed an interesting method of computing stresses in tunnel linings. However, the conclusion that linings designed by current methods are too heavy seems premature. Actually, the tunnel lining carries a more complicated loading than that described by Mr. Bull. The final shape of a tunnel lining often is established after pressure grouting was applied. For example, the Posey Tube[33] in Oakland, Calif., was moved laterally by the forces transmitted through the backfill. The final stresses in the tunnel lining are also influenced by the loading on the roadway slab and by the forces transmitted through the tie rods. These influences were not mentioned by the author.

The coefficient of compressibility, K, has been used widely in the past. Recent experiments, however, have revealed that this coefficient is variable. Therefore, it cannot be used in computing stresses by the principle of superposition of stresses and deformation.

The design loading to be adopted is often influenced by the method of erection of structure. In Section 29, Mr. Bull has stated that no other type of structure requires that erection material be made an integral part, as in tunnel lining. Unfortunately, extra material must be provided for erection stresses in several other types. Pre-cast tunnel tubes are proportioned for the maximum stresses that may be developed during the launching and while shipping the tube sections to their final location. The cantilever method of erecting continuous bridge trusses requires that various members and connections, that are not fully stressed in the completed structure, be reinforced to

Note.—This paper by Anders Bull was published in November, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1945, by J. A. Van den Broek; February, 1945, by Nathan D. Brodkin, M. A. Drucker, and Sigvald Johannesson; May, 1945, by Jacob Feld; and June, 1945, by Leon Beskin, and D. P. Krynine.

[32] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[32a] Received by the Secretary June 18, 1945.

[33] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 747.

resist erection stresses. Extra heavy, reinforcing armature sometimes is provided in reinforced concrete arches to support the centering.

Mr. Bull believes it is erroneous to assume that the vertical soil reactions are uniformly distributed along a horizontal diameter of tube (see Section 19). The accuracy of this assumption varies with the ratio of the horizontal soil pressure to the vertical soil pressure. When this ratio is approaching unity, the error of the adopted assumption is negligible. The assumption of uniform distribution of the vertical reactions simplifies the computation of stresses in the tunnel lining.[34] Of course, deviation from this assumption is quite common when local conditions require it—for instance, when the tube is supported on a cushion or on piers. When the ratio of horizontal soil pressure to vertical soil pressure is less than unity, a simplified graphical expression of soil pressure exerted by a prism is generally adopted.[35] Such a diagrammatic expression of loading gives a convenient criterion for the stresses in the lining. The diagrammatic loading is easy to visualize. It can be compared with the actual loading encountered during erection and, later, when the structure is carrying the actual superimposed load.

---

[34] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 271.

[35] *Ibid.*, Vol. 95 (1931), p. 376.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONVERSION OF KINETIC TO POTENTIAL ENERGY IN FLOW EXPANSIONS

Discussion

BY A. R. THOMAS

A. R. THOMAS,[42] ESQ.[42a]—The need for data on the characteristics of turbulence in flow expansions has long been evident and Professor Kalinske's observations and analysis are of great interest and value.

The data presented refer to circular pipes, but the problem of designing expansions in open channels is even more complex and is of considerable importance where the channel downstream of the expansion is erodible, because a poor design often results in the creation of a return eddy which plays havoc with the side banks. The main principles are the same in open channels as in circular pipes, and, although there are added complexities due to cross-sectional shape and a free water surface, many of the conclusions drawn for the one case should be applicable to the other.

In the experiments described, the flow appears to have been symmetrical about the central axis. The existence of an adverse pressure gradient in the expansion, however, may result in instability of flow pattern, due to which the main stream moves to one side, with the formation of slack water or a return eddy on the other, as shown in Fig. 8.

FIG. 8.

The maintenance of the return eddy depends on a balance of forces which may be expressed in a simplified form as follows:[43] Because of the conversion of kinetic energy into pressure energy, the pressure is greater at point A than at point B, creating conditions favorable to flow from points A to B. Considering the eddy as a whole, its equilibrium is maintained by the balance between the force due to this pressure gradient, acting upstream, the force due to the

NOTE.—This paper by A. A. Kalinske was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by F. T. Mavis; May, 1945, by J. C. Stevens; and June, 1945, by Boris A. Bakhmeteff, and E. R. Van Driest.

[42] Secretary, Central Board of Irrig., Simla, India.

[42a] Received by the Secretary June 27, 1945.

[43] "Flow in Expansions in Open Channels," by A. R. Thomas, *Proceedings*, Punjab Eng. Cong., Lahore, Paper No. 236, 1940, p. 179.

lateral influx of momentum from the main stream between points A and B, due to turbulence, acting downstream, and the resistance of the side wall and bed, acting against the flow. Local pressure differences are created by the curvature of flow, determined to some degree by the shape of expansion, and the shape of eddy adjusts itself to maintain the balance.

The influx of momentum has greatest effect on the outside of the eddy, decreasing toward the side wall. If its effect at point D, the center of the eddy, is not sufficient to outbalance the adverse pressure gradient, flow at that point will be upstream; and the width of eddy will increase, contracting the main stream, raising its velocity, and so increasing the influx of momentum until equilibrium is reached. On the other hand, if the influx more than outbalances the adverse pressure gradient at point D, flow at that point will be downstream, and the eddy will contract until equilibrium is reached. If the influx of momentum is so great that it outbalances the adverse pressure gradient even at the side wall, the eddy will disappear and the entire flow will be in the downstream direction.

This is another way of describing how the lateral transmission of momentum determines the velocity distribution, but it facilitates the drawing of certain conclusions; namely: (1) The greater (*a*) the angle of divergence, (*b*) the expansion ratio, (*c*) the depth-width ratio, or (*d*) the Froude number $(U^2/\sqrt{2\,g\,D})$, the greater is the pressure gradient—hence the larger is the eddy; and (2) the greater the turbulence, the greater is the influx of momentum—hence the smaller is the eddy.

As conclusion (1) assumes turbulence to be independent of the variables considered (which, as shown in the paper, is not generally true) it may have limitations. Conclusion (2) explains why measures taken to increase turbulence (such as a cross grid, or blocks on the bed) improve the flow. The effect of such measures is increased by the loss of head caused, which reduces the pressure gradient. It also explains why, turbulence being subject to scale effect, models of flow expansions do not accurately represent larger prototypes.

It is also clear that, in addition to the angle of divergence, the expansion ratio, depth : width ratio, Froude number, and intensity of turbulence determine the behavior of expanding flow, so that results obtained with a given angle of divergence cannot be applied indiscriminately to other cases.

These conclusions, however, are only qualitative and do not give much assistance in practical design. For example, they do not indicate the maximum permissible angle of expansion in a given case or the most suitable side curves. A quantitative expression in terms of the discharge and channel dimensions can be derived for pressure gradient, modified by loss of head based on empirical data, but it has not so far been possible to derive an expression for the lateral transfer of momentum due to turbulence in expanding flow. It is here that experimental and analytical work, such as is presented by Professor Kalinske, will be of value, and it is hoped that he will be able to extend his researches to other cases.